Lecture

In the last lecture I will give an overview of the course and answer questions about the curriculum and the exam.

Exercises

Independent set and Clique Consider the following two problems:

Clique
Input: An undirected graph $G = (V, E)$ and a natural number k.
Output: YES if the graph G has a k-clique, and NO otherwise.
For an undirected graph $G = (V, E)$ a clique is a subset $V' \subseteq V$ of the vertices such that all vertices in V' are neighbors, i.e., for all $v, w \in V', y \neq w : (v, w) \in E$. We say that G has a k-clique if $|V'| = k$.

Independent Set
Input: An undirected graph $G = (V, E)$ and a natural number k.
Output: YES if the graph G has an independent set of size k, and NO otherwise.
For an undirected graph $G = (V, E)$ a independent set is a subset $V' \subseteq V$ of the vertices such that no two vertices of V' has an edge between them, i.e.,for all $v, w \in V', y \neq w : (v, w) \notin E$.

Q1 Show that Independent set is in NP by describing a polynomial time algorithm to verify a proposed solution. That is, explain which certificate/proposed solution you use, and how it can be verified.

Q2 Clique is a NP-complete problem. Show that independent set is NP-complete by giving a reduction from Clique.

Hamiltonian Path Consider the following two problems:

Hamiltonian Path
Input: An undirected graph $G = (V, E)$ and two vertices u and v from V.
Output: YES if the graph G has a Hamiltonian path, and NO otherwise.
A Hamiltonian path is a **simple** path that visits every vertex exactly once.

Hamiltonian Cycle
Input: An undirected graph $G = (V, E)$.
Output: YES if the graph G has a Hamiltonian cycle, and NO otherwise.
A Hamiltonian cycle is a **simple** cycle that contains every vertex of the graph exactly once.
Deciding wether a graph has a Hamiltonian cycle is a NP-complete problem.

Q1. Hamiltonian path in NP Show that Hamiltonian path is in NP by describing a polynomial time algorithm to verify ta proposed solution. That is, explain which certificate/proposed solution you use, and how it can be verified.

Q2. Hardness of Hamiltonian path Use a reduction from Hamiltonian cycle to show that Hamiltonian path is NP-complete.

Q3. Longest path The shortest path problem between two vertices s and t in a weighted graph can be solved in polynomial time. In the longest path problem we want to find the longest simple path between two vertices s and t in a weighted graph with non-negative edge weights. The length of a path is the sum of the edge weights of the edges on the path.

Formulate a decision version of the longest path problem and show that it is NP-complete (hint: use a reduction from Hamiltonian path).

Exam F14 Solve Question 6 in the exam set from Fall 2014.

CodeJudge Solve the exercises on CodeJudge you haven't solved yet.