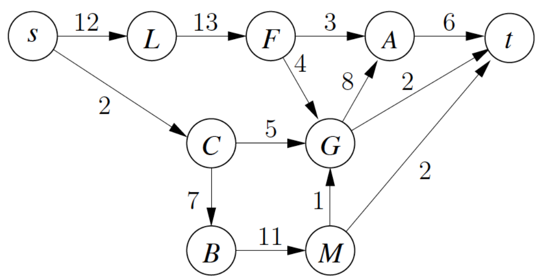
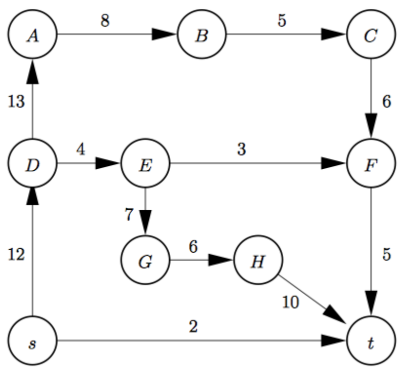
Lecture At the lecture we will talk about string matching algorithms: the string matching automaton and the Knuth-Morris-Pratt algorithm (KMP). You should read CLRS section 32.0, 32.3, 32.4.

Exercises

The Edmonds-Karp algorithm Use Edmonds-Karp's algorithm to compute a maximum flow and minimum cut on the two graphs below. For each augmenting path write the nodes on the path and the value you augment the path with.





Blood donors At the halloween party at a well-known academic institution north of Copenhagen not all went smooth and some students had to be taken to medical emergency treatment at *Rigshospitalet*. In total 150 had to get a transfusion of one bag of blood. The hospital had 155 bags in stock. The distribution of blood groups in the supply and amongst the students is shown in the table below.

Blood type	A	B	0	AB
Bags in stock	44	31	42	38
Demand	37	33	40	40

Type **A** patients can only receive blood of type **A** or type **0**; type **B** patients can receive only type **B** or type **0**; type **0** patients can receive only type **0**; and type **AB** patients can receive any of the four types.

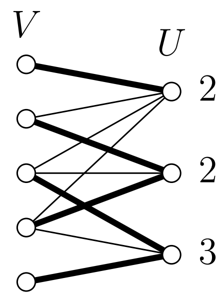
Model the problem as a flow network problem. Draw the corresponding network, and interpret the meaning of the nodes, and edges (edges capacities). Describe how to check whether every student can get a transfusion, otherwise how many can get one.

You do not have to solve the problem explicitly.

Incrementing capacity Suppose you are given a directed graph $G = (V, E)$, with integer capacities on the edges, a source node $s \in V$ and a sink node $t \in V$, and a maximal $s - t$ flow in G. Now suppose we increment the capacity of one of the edges $e \in E$ by one. Give an algorithm to find a maximal flow in the resulting graph.

Generalized matching Consider an undirected bipartite graph $G = (V \cup U, E)$. We want to find a maximum generalized matching M (subset of the edges E) between vertices in V and U, such that each vertex in v is matched to at most one vertex in U, and a vertex u in U is matched to at most d_u vertices i V. For each node u in U, the upper bound d_u is given as part of the input.

The example below shows an example of a generalized matching of size 5 (the upper bounds are written next to the vertices).

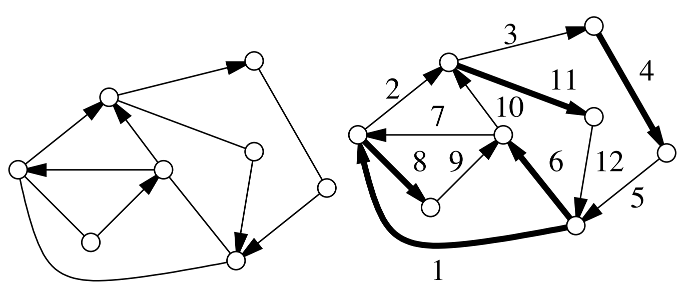


Give an algorithm to find a maximal generalized matching.

Euler tours in mixed graphs It is well-known that a strongly connected graph has an Euler tour (a cycle that contains each edge exactly once) if and only if any vertex have the same number of ingoing and outgoing edges.

In the following we consider *mixed graphs*, which are graphs where some edges are directed and some are not. We assume that the graphs are connected, i.e., if we ignore the orientation on all directed edges, then the graph is connected.

We want an algorithm to decide wether it is possible to assign directions to all undirected edges, such that the graph has an Euler tour. In the example below the mixed graph on the left can be assigned directions (graph on right) such that there is an Euler tour (the numbers on the edges denotes the order the edges can be visited in).



Give an algorithm to decide wether it is possible to assign directions to all undirected edges, such that a graph G has an Euler tour.

Hint: Consider a bipartite graph, where the nodes on one side are edges in G and the nodes on the other side are the vertices in G. Use your algorithm from the previous exercise.

Puzzle of the week: The twelve-coin problem There are twelve coins, eleven of which are identical and one of which is different, but it is not known whether it is heavier or lighter than the others. You have a traditional balance scale with two pans. To use such a scale, you place a coin into each pan and the scale will determine which coin is heavier. The balance or scale may be used three times to isolate the unique coin and determine wether it is heavier or lighter than the others.

Mandatory: Christmas trees (from the exam E15) The Dean has asked you to arrange the annual Christmas party for the students at DTU. You have to make a plan for how to place the tables in the hall. The local fire department has divided the hall up into an $n \times m$ grid of subsquares and declared that you can place at most two tables in each row and at most one in each column. Unfortunately, the Dean who loves Christmas has put up Christmas trees in many of the subsquares. You cannot place a table in a subsquare with a Christmas tree.

Example Here $n = 4$ and $m = 8$. The $*$ are Christmas trees and T are tables. In the example the maximum number of tables that can be placed is 7.

$*$	T						T
T	$*$	$*$	T	$*$	$*$		$*$
	$*$	$*$		$*$	$*$	T	$*$
	$*$	T	$*$		T	$*$	

Question 1 Model the problem as a graph problem. Explain how you model the problem as a graph problem and draw the graph corresponding to the example above.

Question 2 Describe an algorithm that given n, m, and the placement of the Christmas trees computes the maximum number of tables you can place in the hall. Analyze the asymptotic running time of your algorithm. Remember to argue that your algorithm is correct.