Reading Material

At the lecture we will talk more about network flow. We will consider two applications: bipartite matching and finding disjoint paths. We will also discuss how to find good augmenting paths. You should read KT 7.3, 7.5 and 7.6.

Exercises

1 The Edmonds-Karp algorithm and the scaling algorithm Use both the Edmonds-Karp’s algorithm and the scaling algorithm to compute a maximum flow and minimum cut on the two graphs below. For each augmenting path write the nodes on the path and the value you augment the path with.

2 Blood donors At the halloween party at a well-known academic institution north of Copenhagen not all went smooth and some students had to be taken to medical emergency treatment at Rigshospitalet. In total 150 had to get a transfusion of one bag of blood. The hospital had 155 bags in stock. The distribution of blood groups in the supply and amongst the students is shown in the table below.

<table>
<thead>
<tr>
<th>Blood type</th>
<th>A</th>
<th>B</th>
<th>0</th>
<th>AB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bags in stock</td>
<td>44</td>
<td>31</td>
<td>42</td>
<td>38</td>
</tr>
<tr>
<td>Demand</td>
<td>37</td>
<td>33</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

Type A patients can only receive blood of type A or type 0; type B patients can receive only type B or type 0; type 0 patients can receive only type 0; and type AB patients can receive any of the four types.

Model the problem as a flow network problem. Draw the corresponding network, and interpret the meaning of the nodes, and edges (edges capacities). Describe an algorithm to check whether every student can get a transfusion, otherwise how many can get one.

You do not have to solve the problem explicitly.
3 **Generalized matching**  Consider an undirected bipartite graph $G = (V \cup U, E)$. We want to find a maximum generalized matching $M$ (subset of the edges $E$) between vertices in $V$ and $U$, such that each vertex in $v$ is matched to at most one vertex in $U$, and a vertex $u$ in $U$ is matched to at most $d_u$ vertices in $V$. For each node $u$ in $U$, the upper bound $d_u$ is given as part of the input.

The example below shows an example of a generalized matching of size 5 (the upper bounds are written next to the vertices).

![Graph Example](image)

Give an algorithm to find a maximal generalized matching.

4 **Ergonomic floor plans**  Solve exercise KT 7.6

5 **Escape**  Solve exercise KT 7.14.

**Puzzle of the week: The twelve-coin problem**  There are twelve coins, eleven of which are identical and one of which is different, but it is not known whether it is heavier or lighter than the others. You have a traditional balance scale with two pans. To use such a scale, you place a coin into each pan and the scale will determine which coin is heavier. The balance or scale may be used three times to isolate the unique coin and determine whether it is heavier or lighter than the others.