

Technical University of Denmark

Example exam.

Course name: Algorithms and data structures.

Course number: 02110.

Aids allow: All written materials are permitted.

Exam duration: 4 hours

Weighting: Question 1: 30% - Question 2: 15% - Question 3: 20% - Question 4: 10% - Question 5: 15% - Question 6: 10%.

The weighting is only an approximative weighting.

All questions should be answered by filling out the room below the question. As exam paper just hand in this and the following pages filled out. If you need more room you can use extra paper that you hand in together with the exam paper.

Question 1

1.1 Which of the following statements are correct: :

- A The worst-case running time of the Jarvis' March algorithm is $O(n^2)$.
- B Jarvis' March is always faster than Graham's scan.
- C If the points are sorted by polar angles then Jarvis' march runs in linear time.
- D Graham's scan is always faster than Jarvis' March.

1.2 Which of the following statements are true: :

- A A subtree of a red-black tree is itself a red-black tree (except the root might be red).
- B The sibling of a leaf node is either a leaf or red.
- C the *longest* simple path from a node x in a red-black tree to a descendant leaf has length at most twice that of the *shortest* path from x to a descendant leaf.

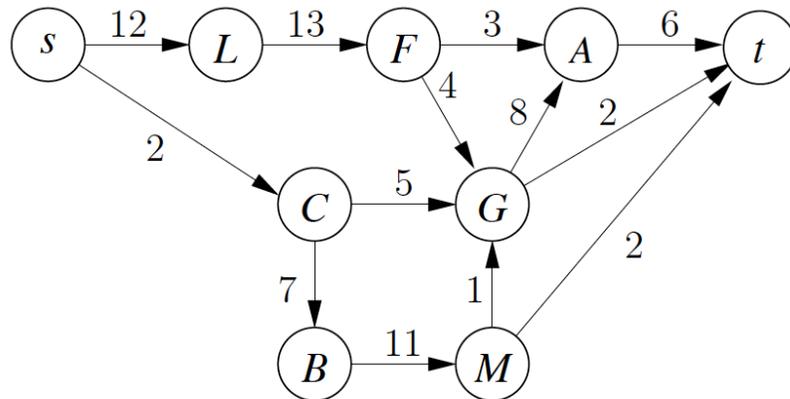
1.3 Draw the compressed trie for the strings **this**, **that**, **hat**, **thing**, **fat** (don't replace the labels by indexes into the string, just write the labels on the edges):

1.4 Draw the finite string matching automata for the string `hejhejsa`:

1.4 Show the red-black tree that results from inserting the keys 41, 38, 31, 12, 19 , 8 in this order into an initially empty tree.

Question 2 (flow)

Consider the network below with capacities on the edges.



Question a Give a maximum flow from s to t in the network (write the flow for each edge along the edges on the graph above), give the value of the flow, and give a minimum $s - t$ cut (give the partition of the vertices).

value of flow: _____

minimum cut: _____

Question b Use Edmonds-Karp's algorithm to compute a maximum flow on the two graph. For each augmenting path write the nodes on the path and the value you augment the path with in the table below.

augmenting path	value

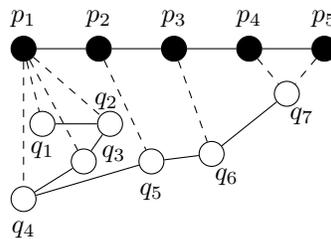
Question 3

Consider Professor Bille going for a walk with his personal zombie. The professor follows a path of points p_1, \dots, p_n and the zombie follows a path of points q_1, \dots, q_m . We assume that the walk is partitioned into a number of small steps, where the professor and the zombie in each step either both move from p_i to p_{i+1} and from q_j to q_{j+1} , respectively, or only one of them moves and the other one stays.

The goal is to find the smallest possible length L of the leash, such that the professor and the zombie can move from p_1 and q_1 , resp., to p_n and q_n . They cannot move backwards, and we only consider the distance between points. The distance L is also known as the discrete Fréchet distance.

We let $L(i, j)$ denote the smallest possible length of the leash, such that the professor and the zombie can move from p_1 and q_1 to p_i and q_j , resp. For two points p and q , let $d(p, q)$ denote the distance between them.

In the example below the dotted lines denote where Professor Bille (black nodes) and the zombie (white nodes) are at time 1 to 8. The minimum leash length is $L = d(p_1, q_4)$.



Q1: Recurrence Give a recursive formula for $L(i, j)$.

Q2: Algorithm Give an algorithm that computes the length of the shortest possible leash. Analyze space and time usage of your solution.

Q3: Print solution Extend your algorithm to print out paths for the professor and the zombie. The algorithm must return where the professor and the zombie is at each time step. Analyze the time and space usage of your solution.

Question 4

At the halloween party at a well-known academic institution north of Copenhagen not all went smooth and some students had to be taken to medical emergency treatment at *Rigshospitalet*. In total 150 had to get a transfusion of one bag of blood. The hospital had 155 bags in stock. The distribution of blood groups in the supply and amongst the students is shown in the table below.

Blood type	A	B	0	AB
Bags in stock	44	31	42	38
Demand	37	33	40	40

Type **A** patients can only receive blood of type **A** or type **0**; type **B** patients can receive only type **B** or type **0**; type **0** patients can receive only type **0**; and type **AB** patients can receive any of the four types.

Model the problem as a flow network problem. Draw the corresponding network, and interpret the meaning of the nodes, and edges (edges capacities). Describe how to check whether every student can get a transfusion, otherwise how many can get one.

You do not have to solve the problem explicitly. Remember to argue that your algorithm is correct.

Question 5

In chemical databases for circular molecules, each molecule is represented by a circular string of chemical characters. To allow faster lookup and comparisons of molecules, one wants to store each circular string by a canonical linear string. A natural choice for a canonical linear string is the one that is lexicographically smallest. That gives the following computational problem.

Assume we are given a string $T = x_1 \dots x_n$ of length n . A *shift* of T by s , $0 \leq s < n$, is the string $T^s = x_{s+1}x_{s+2} \dots x_n x_1 x_2 \dots x_s$. In this problem we want to find the *lexicographically smallest shift*, i.e. the shift s where T^s is lexicographically smallest among T^0, \dots, T^{n-1} . Eg. $T^2 = T^7 = \mathbf{a a b a b a a b a b}$ are the lexicographically smallest shifts of the string

$$T = \mathbf{a b a a b a b a a b}$$

Question a State all s where T^s is a lexicographically smallest shift of the string

$$T = \mathbf{b c a b a a b c a b a a b c a b a a}$$

Question b Describe an algorithm that given a string T of length n over an alphabet of size $O(1)$ computes all s where T^s is a lexicographically smallest shift of T . State the algorithm's running time.

Question 6

Let S be a set of n line segments in the plane. Give an algorithm to compute the convex hull of S . Analyze the time complexity of your algorithm and argue it is correct.