String Dictionaries: Tries

Inge Li Gørtz

String Dictionaries

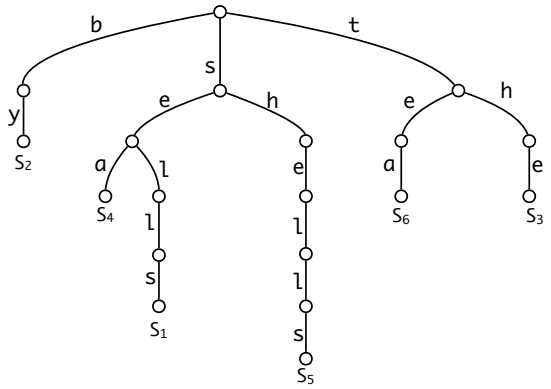
- String dictionary problem. Let $S = \{S_1, S_2, \ldots, S_k\}$ be a set of strings of characters from alphabet Σ. Preprocess S into data structure to support:
  - search(P): Return yes if $P = S_i$ for some S_i in S.
  - prefix-search(P): Return all strings in S for which P is a prefix.

- Example. S = {sells, by, the, sea, shells, tea}.
  - prefix-search('se') returns 'sea' and 'sells'.

Tries

Tries

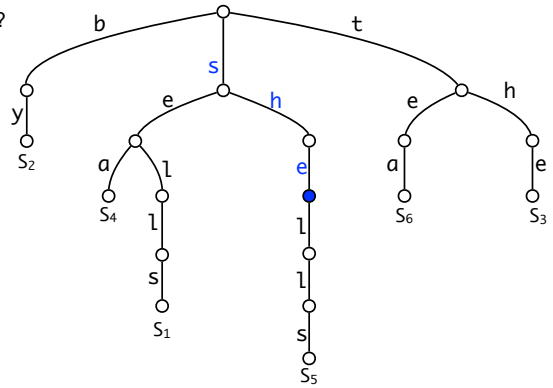
- Text retrieval



- Trie over the strings: sells, by, the, sea, shells, tea.

Tries

- Text retrieval
- Prefix-free?



- Trie over the strings: sells, by, the, sea, shells, tea, she.

Tries

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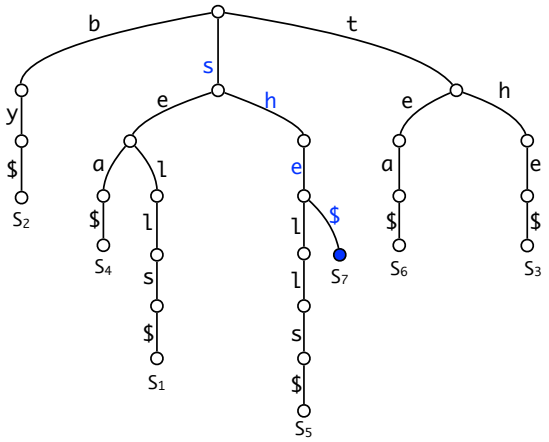
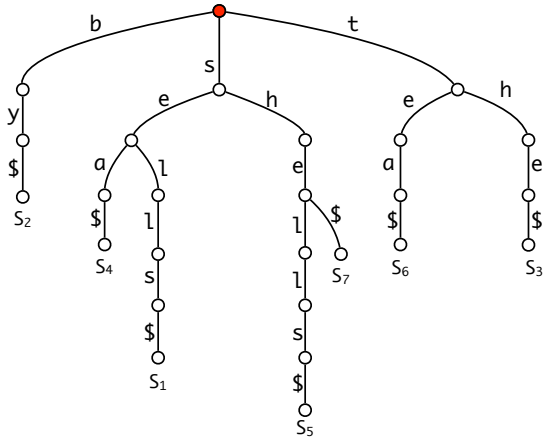


- Trie over the strings: sells$, by$, the$, sea$, shells$, tea$, she$.

Tries: Searching

- Search for ‘sea$’:



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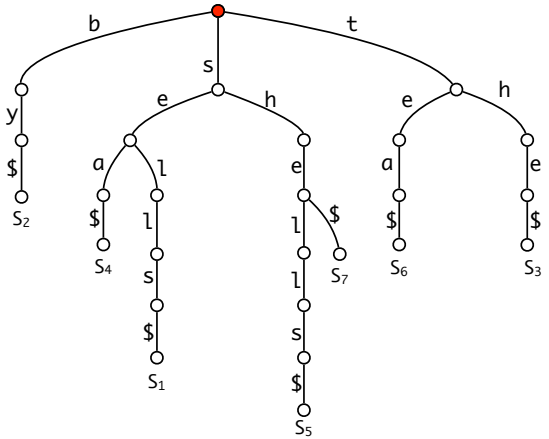

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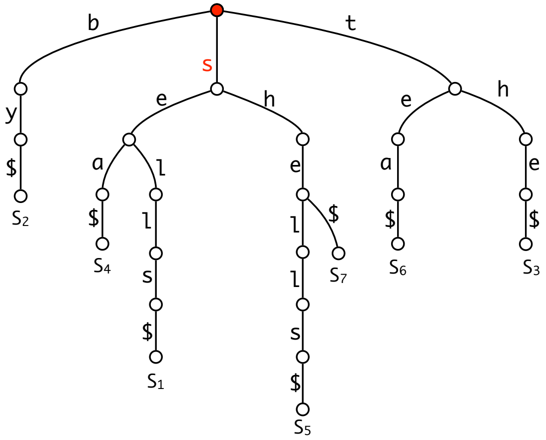

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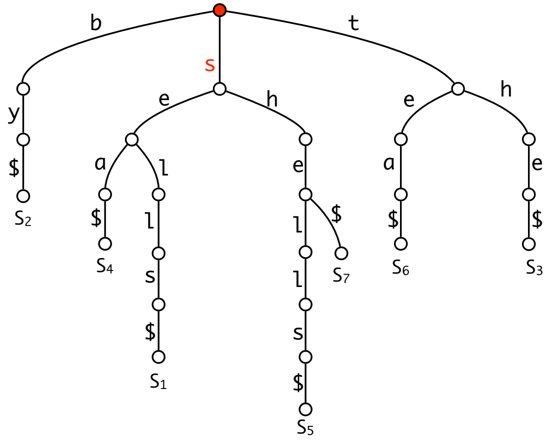

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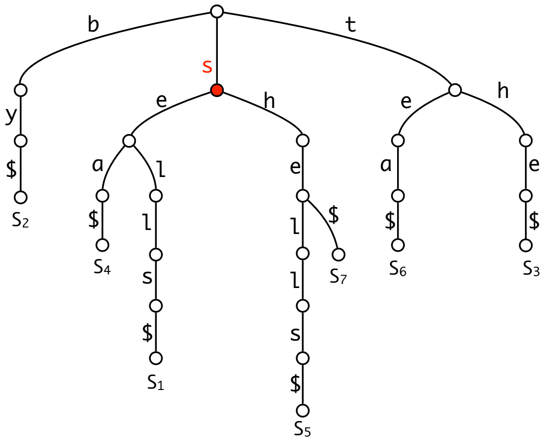

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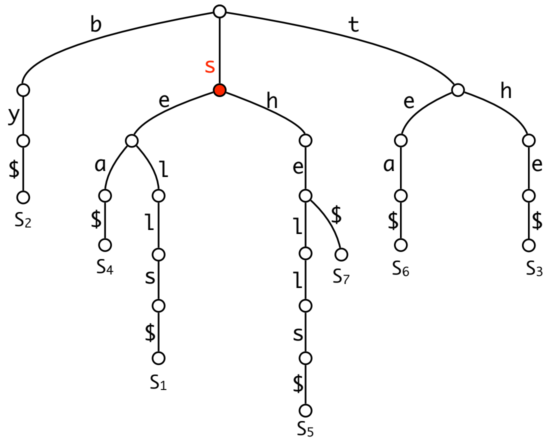

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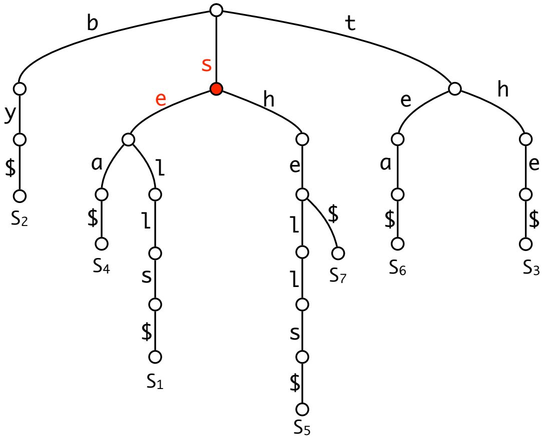

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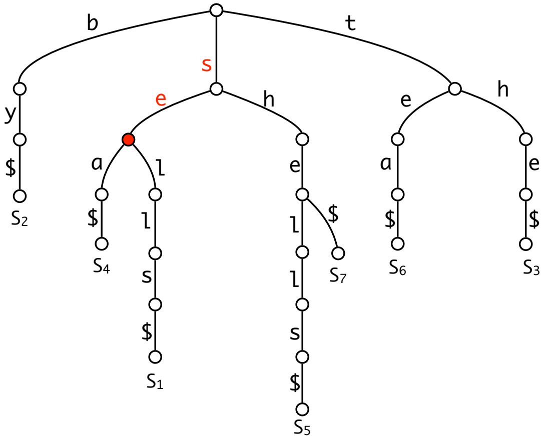

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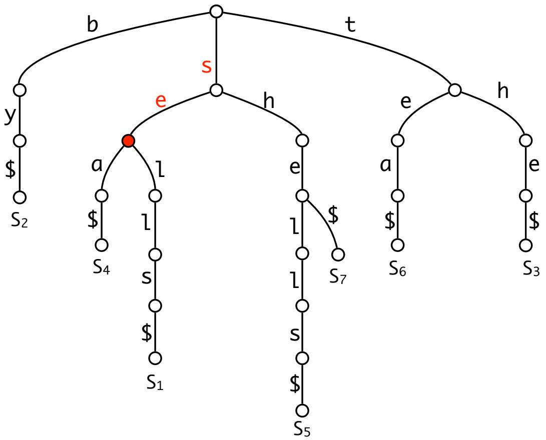

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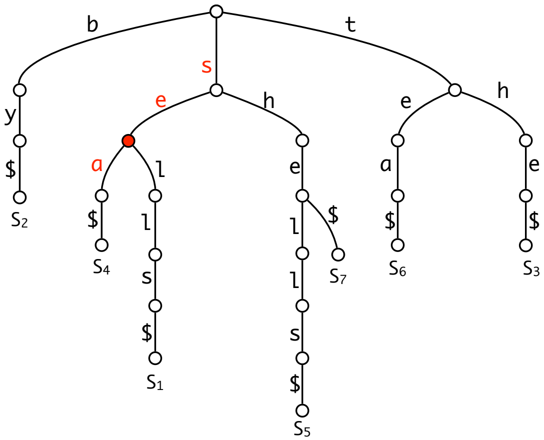

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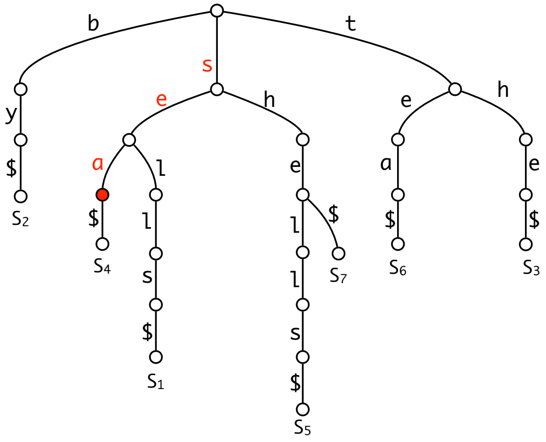

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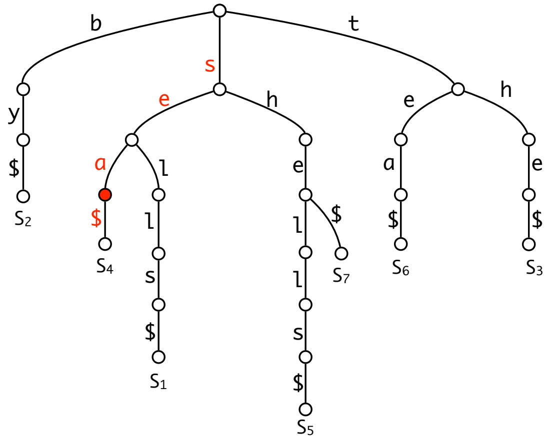

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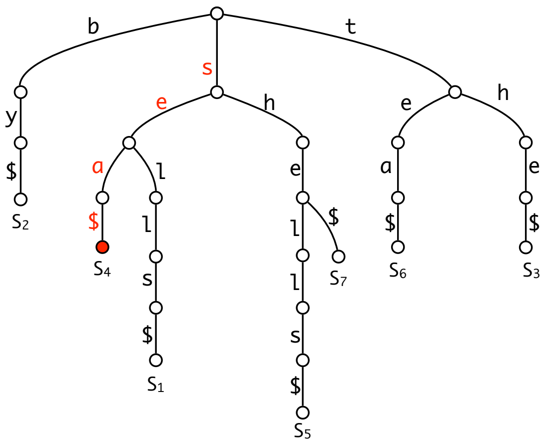

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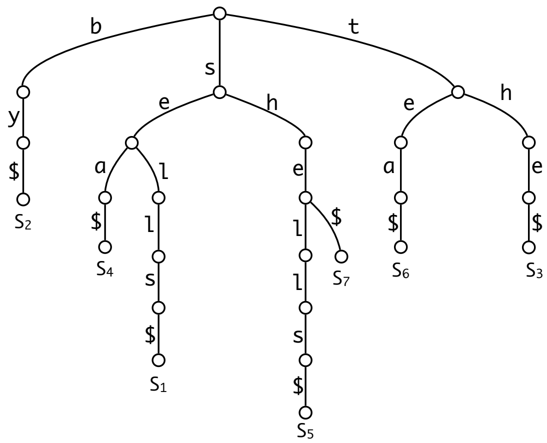


Tries

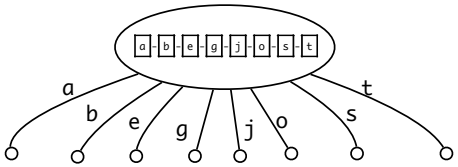
- Fredkin 1960]. Retrieval. Store a set of strings in a rooted tree such that:
  - Each edge is labeled by a character. Edges to children of a node are sorted from left-to-right alphabetically.
  - Each root-to-leaf path represents a string in the set. (obtained by concatenating the labels of edges on the path).
  - Common prefixes share same path maximally.

Trie

- Properties of the trie. A trie T storing a collection S of s strings of total length *n* from an alphabet of size *d* has the following properties:
  - How many children can a node have? at most d
  - How many leaves does T have? s
  - What is the height of T? length of longest string
  - What is the number of nodes in T? O(n)

Trie

- Search time:
  - List: O(d) in each node => O(dm).
    - O(m) if d constant.



Trie

- Search time:
  - List: O(d) in each node => O(dm).
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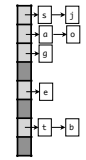
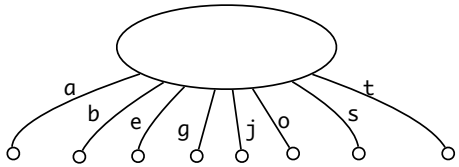
  - d not constant: use dictionary
    - Hashtable with universal hashing: expected O(1)





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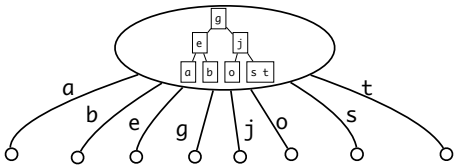
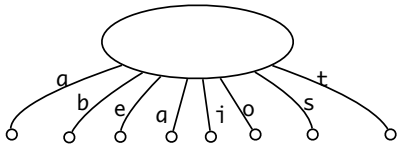
    - Balanced Search Tree: O(log d)



Trie

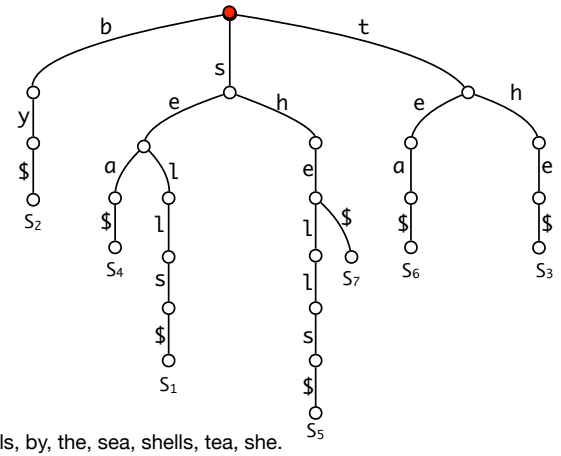
- Search time:
  - List: O(d) in each node => O(dm).
    - O(m) if d constant.
  - d not constant: use dictionary
    - Hashtable with universal hashing O(1)
    - Balanced BST: O(log d)
- Space for a trie:
  - O(n) nodes and edges.
  - Space for data structure in each node:
    - If each node uses space proportional to the number of edges out of it, then

$$\text{space} = \sum_v \text{size of data structure} = \#\text{edges} = O(n).$$



Construction of Tries

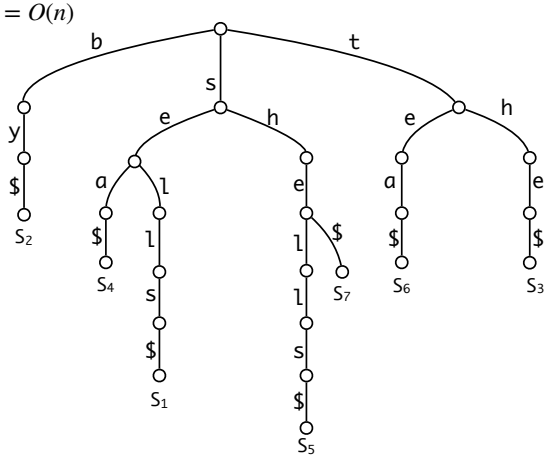
- Insert the strings one-by-one.



- Trie over the strings: sells, by, the, sea, shells, tea, she.

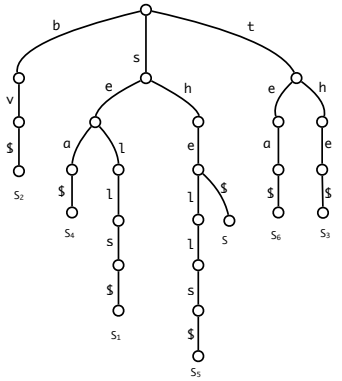
Construction of Tries

- Insert the strings one-by-one.
  - $O(n_i)$ for for string of length n_i.
  - Total time: $O(\sum_i n_i) = O(n)$



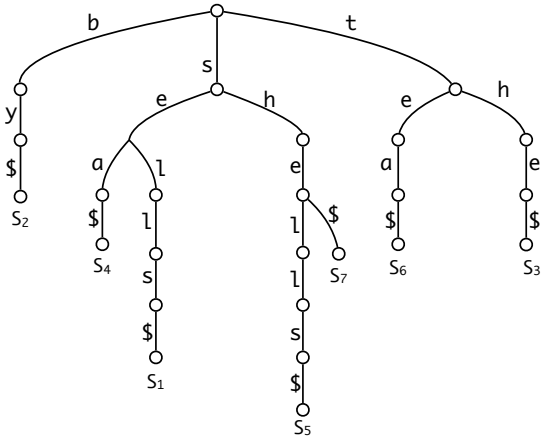
Trie

- Time and space for a trie (for constant d):
  - O(m) for searching for a string of length m.
  - O(n) space.
  - Preprocessing: O(n)



Prefix search

- Prefix search: return all words in the trie starting with “se”



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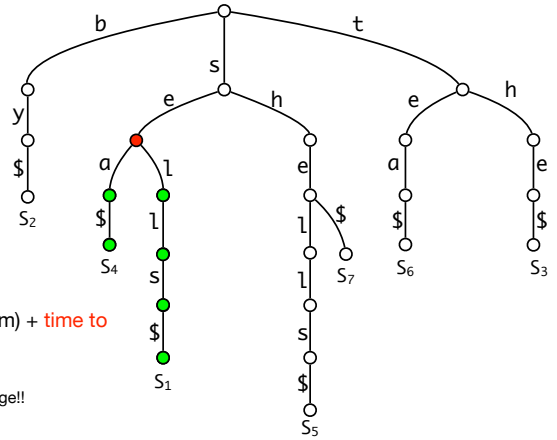


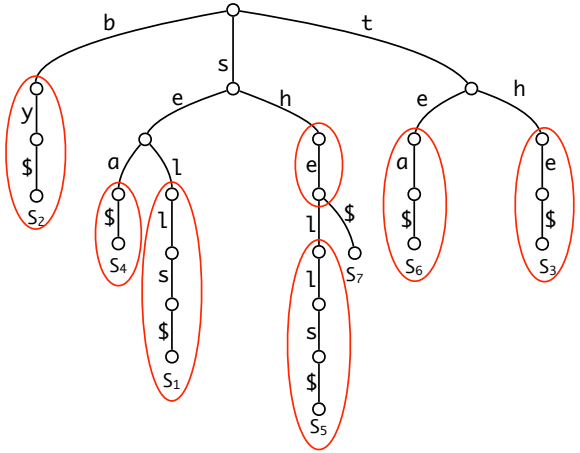
- Time for prefix search: O(m) + time to report all occurrences.

Could be large!!

- Want: O(m) + O(number of occurrences)

Tries

- Compact trie. Chains of nodes with a single child is merged into a single edge.



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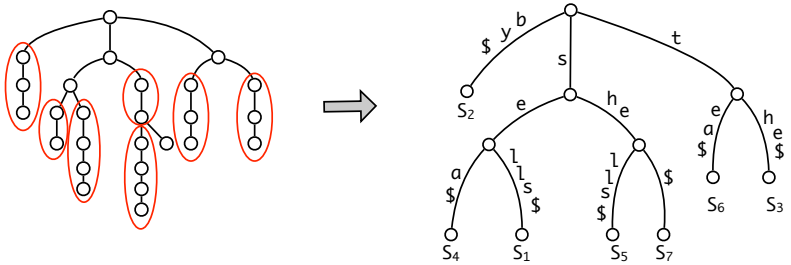
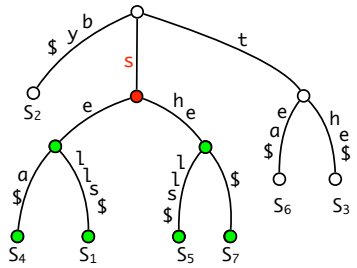


- Properties of the compact trie. A compact trie T storing a collection S of s strings of total length *n* from an alphabet of size *d* has the following properties:
  - Every internal node of T (except possibly the root) has at least 2 and at most *d* children.
  - T has *s* leaves
  - The number of nodes in T is ≤ 2s.

Prefix Search in Compact Trie

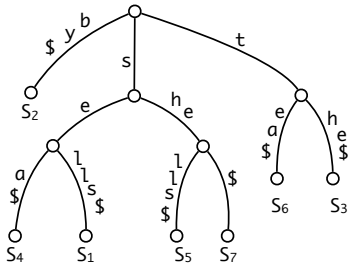
- Prefix search: return all words in the trie starting with “s”

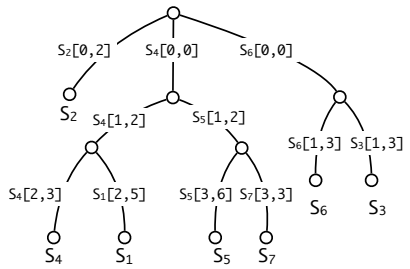


- Time for prefix search: O(m) + time to report all occurrences. = O(m + number of occurrences)

Storing the Compact Trie

- If we store all edge labels explicitly, we still use O(n) space.
- Instead store edge labels as reference into the strings (strings are stored).

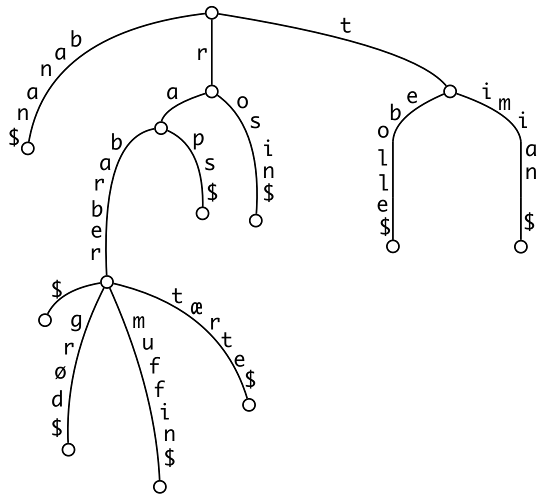




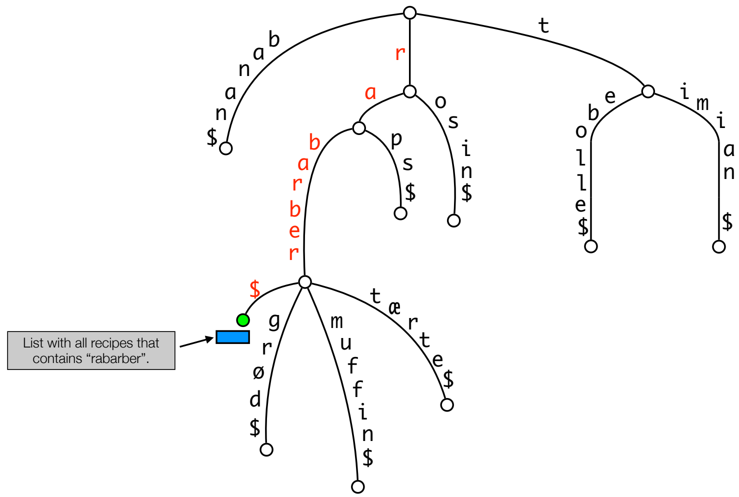
Trie

- Time and space for a compact trie of s strings of total length n over a constant size alphabet.
  - O(m) for searching for a pattern of length m.
  - O(m + occ) for prefix search, where occ = #occurrences
  - O(s) space (in addition to the input strings).
  - Preprocessing time: O(n)

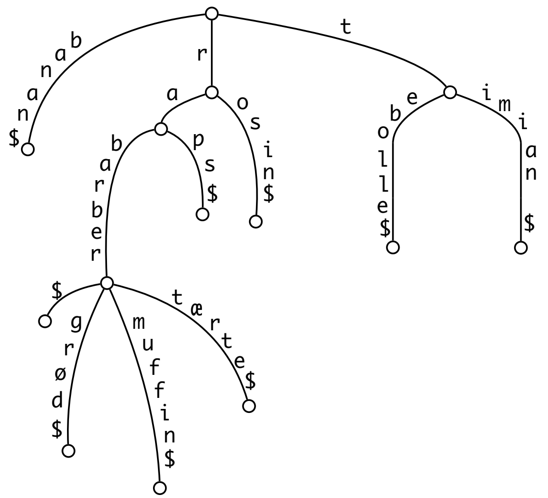
Trie



Inverted Index



Tries: Prefix search



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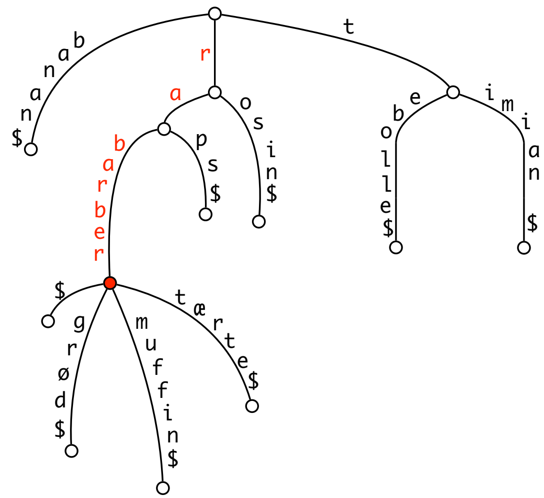

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