Lecture

At the lecture we continue with randomized algorithms. We will talk about hash tables and hash functions.

1 $[w]$ **Run by Hand and Properties**

1.1 Insert the key sequence $K = 7, 18, 2, 3, 14, 25, 1, 11, 12, 1332$ into a hash table of size 11 using chained hashing with hash function $h(k) = k \mod 11$.

1.2 Insert the key sequence $K = 7, 18, 2, 3, 25, 1, 11, 12$ into a hash table of size 11 using linear probing with hash function $h(k) = k \mod 11$.

1.3 Let K be a sequence of keys stored in a hash table A using chained hashing. Given A, can one efficiently find the maximum element in K?

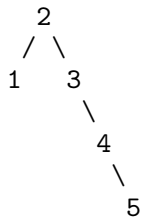
1.4 Show how the hash tables from 1.1 and 1.2 looks after deleting the key 2.

2 Streaming Statistics An IT-security friend of yours wants a high-speed algorithm to count the number of distinct incoming IP-addresses in his router to help detect denial of service attacks. Can you help him?

3 Multi-Set Hashing A multi-set is a set M, where each element may occur multiple times. Design an efficient data structure supporting the following operations:

- ADD(x): Add an(other) occurrence of x to M.
- REMOVE(x): Remove an occurrence of x from M. If x does not occur in M do nothing.
- REPORT(x): Return the number of occurrences of x.

4 Quicksort Consider the following sorting algorithm: Construct a random permutation of the numbers and insert them in this order into an initially empty binary search tree. When all numbers are inserted output the inorder sequence of the numbers. The search tree does not perform any rotations—-to insert a number just search down for to find the correct place and insert it as a leaf. I.e., inserting the numbers $1, 2, 3, 4, 5$ in the order $2, 1, 3, 4, 5$ would give the following search tree:



Prove that the expected running time is $O(n \log n)$. *Hint:* Argue that this is just another way of describing the Quicksort algorithm.

5 Dynamic hashtable Explain how to make a dynamic hashtable with insertions using the doubling technique. Your solution should use $\Theta(n)$ space, where n is the number of elements in the hashtable. What is the insertion time of your solution?

6 The Rabbit Billy (Exam 2017) The rabbit Billy lives with his family in a rabbit hole. The rabbit Billy is very forgetful and he loves carrots. Last week he hid k carrots in k different bushes. Now he is hungry, but he forgot in which of the b bushes around the rabbit hole he hid the carrots. To find a carrot he now does the following. In each round he goes to a random bush and checks if there is a carrot. If not he runs back to the rabbit hole and tries again. Since he is very forgetful he might go to the same bush again in the next round.

6.1 What is the expected number of bushes that Billy visits before he finds the first carrot? Explain your answer.

6.2 After eating the first carrot Billy is still hungry, so he keeps looking for two more carrots. He does not remember where he already has found carrots, so he might go to these bushes again. What is the expected number of bushes that Billy visits before he has found 3 carrots? Explain your answer.