Introduction to Data Structures

- Data structures
- Stacks and Queues
- Linked Lists
- Dynamic Arrays

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Data Structures

- Data structure. Method for organizing data for efficient access, searching, manipulation, etc.
- Goal.
  - Fast.
  - Compact
- Terminology.
  - Abstract vs. concrete data structure.
  - Dynamic vs. static data structure.

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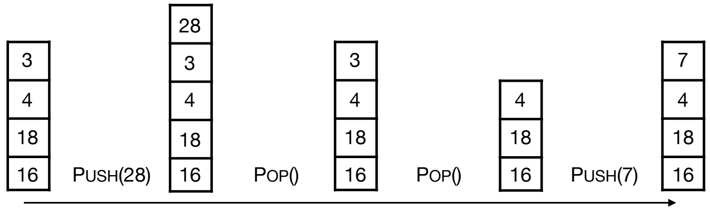
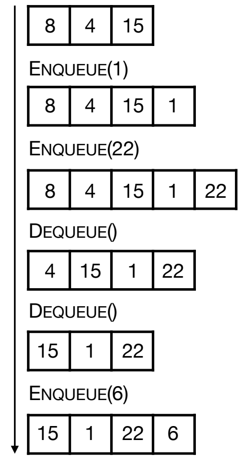
Stack

- Stack. Maintain dynamic sequence (stack) S supporting the following operations:
  - PUSH(x): add x to S.
  - POP(): remove and return the most recently added element in S.
  - ISEMPTY(): return true if S is empty.



Queue

- Queue. Maintain dynamic sequence (queue) Q supporting the following operations:
  - ENQUEUE(x): add x to Q.
  - DEQUEUE(): remove and return the first added element in Q.
  - ISEMPTY(): return true if S is empty.

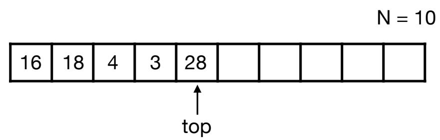


Applications

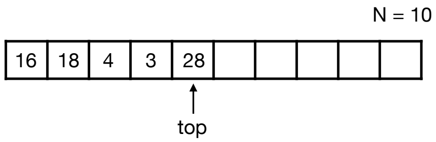
- Stacks.
  - Virtual machines
  - Parsing
  - Function calls
  - Backtracking
- Queues.
  - Scheduling processes
  - Buffering
  - Breadth-first searching

Stack Implementation

- Stack. Stack with capacity N
- Data structure.
  - Array S[0..N-1]
  - Index top. Initially top = -1
- Operations.
  - PUSH(x): Add x at S[top+1], top = top + 1
  - POP(): return S[top], top = top - 1
  - ISEMPTY(): return true if top = -1.
  - Check for overflow and underflow in PUSH and POP.



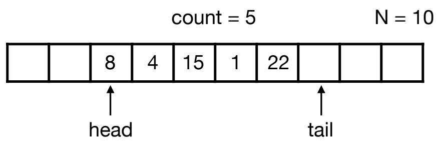
Stack Implementation



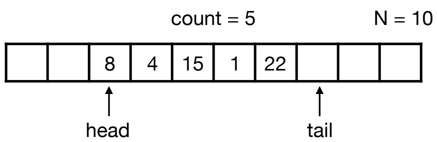
- Time
  - Push in $\Theta(1)$ time.
  - Pop in $\Theta(1)$ time.
  - isEmpty in $\Theta(1)$ time.
- Space.
  - $\Theta(N)$ space.
- Limitations.
  - Capacity must be known.
  - Wasting space.

Queue Implementation

- Queue. Queue with capacity N.
- Data structure.
  - Array Q[0..N-1]
  - Indices head and tail and a counter.
- Operations.
  - Enqueue(x): add x at S[tail], update count og tail cyclically.
  - Dequeue(): return Q[head], update count og head cyclically.
  - isEmpty(): return true if count = 0.
  - Check for overflow and underflow in Dequeue and Enqueue.



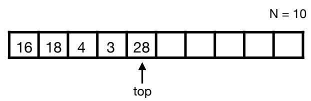
Queue Implementation



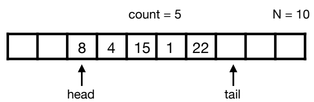
- Time.
  - Enqueue in $\Theta(1)$ time.
  - Dequeue in $\Theta(1)$ time.
  - isEmpty in $\Theta(1)$ time.
- Space.
  - $\Theta(N)$ space.
- Limitations.
  - Capacity must be known.
  - Wasting space.

Stacks and Queues

- Stack.
  - Time. Push, Pop, isEmpty in $\Theta(1)$ time.
  - Space. $\Theta(N)$



- Queue.
  - Time. Enqueue, Dequeue, isEmpty in $\Theta(1)$ time.
  - Space. $\Theta(N)$



- Challenge. Can we get linear space and constant time?

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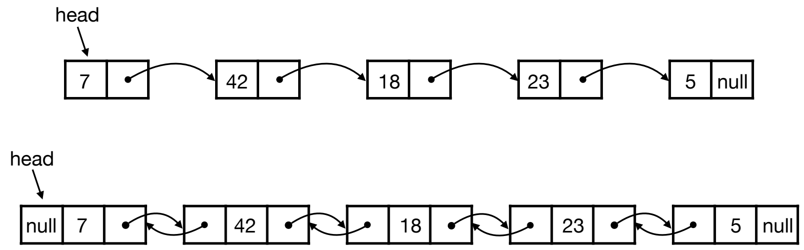
Linked Lists

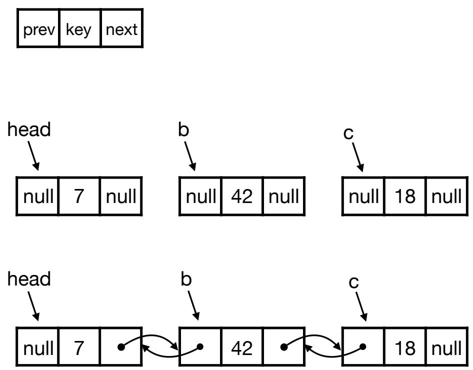
- Linked lists.
  - Data structure to maintain dynamic sequence of elements in linear space.
  - Sequence order determined by pointers/references called links.
  - Fast insertion and deletion of elements and contiguous sublists.
  - Singly-linked vs doubly-linked.



Linked Lists

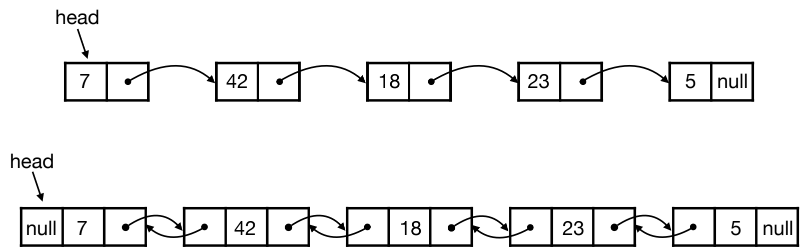
- Doubly-linked lists in Java.

```
class Node {
    int key;
    Node next;
    Node prev;
}

Node head = new Node();
Node b = new Node();
Node c = new Node();
head.key = 7;
b.key = 42;
c.key = 18;

head.prev = null;
head.next = b;
b.prev = head;
b.next = c;
c.prev = b;
c.next = null;
```



Linked Lists

- Simple operations.
  - SEARCH(head, k): return node with key k. Return null if it does not exist.
  - INSERT(head, x): insert node x in front of list. Return new head.
  - DELETE(head, x): delete node x in list.



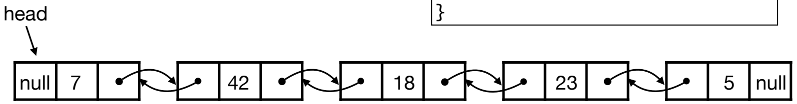
Linked Lists

- Operations in Java.

```
Node Search(Node head, int value) {
    Node x = head;
    while (x != null) {
        if (x.key == value) return x;
        x = x.next;
    }
    return null;
}
```

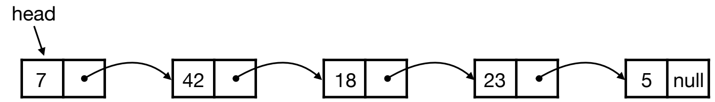
```
Node Insert(Node head, Node x) {
    x.prev = null;
    x.next = head;
    head.prev = x;
    return x;
}
```

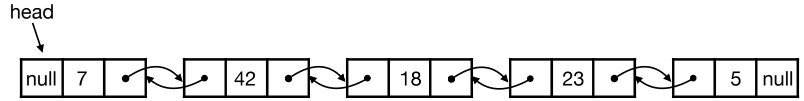
```
Node Delete(Node head, Node x) {
    if (x.prev != null)
        x.prev.next = x.next;
    else head = x.next;
    if (x.next != null)
        x.next.prev = x.prev;
    return head;
}
```



- Ex. Let p be a new with key 10 and let q be node with key 23 in list. Trace execution of `Search(head,18)`, `Insert(head,p)` og `Delete(head,q)`.

Linked Lists





- Time.
  - SEARCH in Θ(n) time.
  - INSERT and DELETE in Θ(1) time.
- Space.
  - Θ(n)

Stack and Queue Implementation

- Ex. Consider how to implement stack and queue with linked lists efficiently.

- Stack. Maintain dynamic sequence (stack) S supporting the following operations:
  - PUSH(x): add x to S.
  - POP(): remove and return the most recently added element in S.
  - ISEMPTY(): return true if S is empty.

- Queue. Maintain dynamic sequence (queue) Q supporting the following operations:
  - ENQUEUE(x): add x to Q.
  - DEQUEUE(): remove and return the first added element in Q.
  - ISEMPTY(): return true if S is empty.

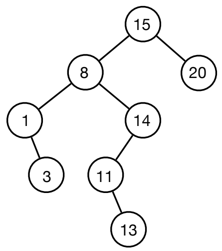
Stack and Queue Implementation

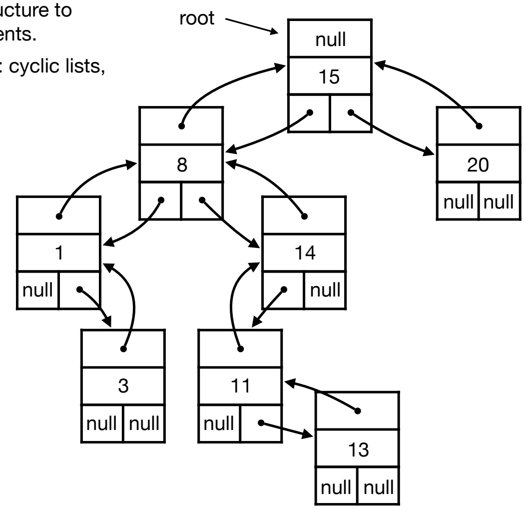
- Stacks and queues using linked lists
- Stack.
  - Time. PUSH, POP, ISEMPTY in Θ(1) time.
  - Space. Θ(n)

- Queue.
  - Time. ENQUEUE, Dequeue, ISEMPTY in Θ(1) time.
  - Space. Θ(n)

Linked Lists

- Linked list. Flexible data structure to maintain sequence of elements.
- Other linked data structures: cyclic lists, trees, graphs, …





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Stack Implementation with Array

- Challenge. Can we implement a stack efficiently with arrays?
  - Do we need a fixed capacity?
  - Can we get linear space and constant time?

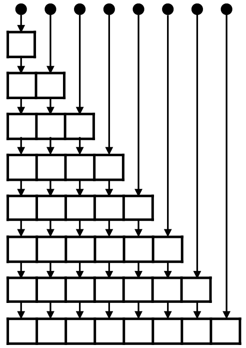
Dynamic Arrays

- Goal.
  - Implement a stack using arrays in $\Theta(n)$ space for n elements.
  - As fast as possible.
  - Focus on PUSH. Ignore POP and ISEMPTY for now.

- Solution 1
  - Start with array of size 1.
- PUSH(x):
  - Allocate new array of size + 1.
  - Move all elements to new array.
  - Delete old array.

Dynamic Arrays

- Push(x):
  - Allocate new array of size + 1.
  - Move all elements to new array.
  - Delete old array.
- Time. Time for n Push operations?
  - ith Push takes $\Theta(i)$ tid.
  - $\Rightarrow$ total time is $1 + 2 + 3 + 4 + \ldots + n = \Theta(n^2)$
- Space. $\Theta(n)$
- Challenge. Can we do better?

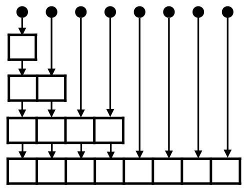


Dynamic Arrays

- Idea. Only copy elements some times
- Solution 2.
  - Start with array of size 1.
- Push(x):
  - If array is full:
    - Allocate new array of twice the size.
    - Move all elements to new array.
    - Delete old array.

Dynamic Arrays

- Push(x):
  - If array is full:
    - Allocate new array of twice the size.
    - Move all elements to new array.
    - Delete old array.
- Time. Time for n Push operations?
  - Push 2^k takes $\Theta(2^k)$ time.
  - All other Push operations take $\Theta(1)$ time.
  - $\Rightarrow$ total time $< 1 + 2 + 4 + 8 + 16 + \ldots + 2^{\log n} + n = \Theta(n)$
- Space. $\Theta(n)$



Dynamic Arrays

- Stack with dynamic array.
  - n Push operations in $\Theta(n)$ time and plads.
  - Extends to n Push, Pop og isEmpty operations in $\Theta(n)$ time.
- Time is amortized $\Theta(1)$ per operation.
- With more clever tricks we can deamortize to get $\Theta(1)$ worst-case time per operation.
- Queue with dynamic array.
  - Similar results as stack.
- Global rebuilding.
  - Dynamic array is an example of global rebuilding.
  - Technique to make static data structures dynamic.

Stack and Queues

Data structure	PUSH	POP	ISEMPTY	Space
Array with capacity N	Θ(1)	Θ(1)	Θ(1)	Θ(N)
Linked List	Θ(1)	Θ(1)	Θ(1)	Θ(n)
Dynamic Array 1	Θ(n)†	Θ(1)†	Θ(1)	Θ(n)
Dynamic Array 2	Θ(1)†	Θ(1)†	Θ(1)	Θ(n)
Dynamic Array 3	Θ(1)	Θ(1)	Θ(1)	Θ(n)

† = amortized

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