

Weekplan: Searching and Sorting

The 02105+02326 DTU Algorithms Team

Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Chapter 2.

Exercises

1 Run by Hand and Properties Solve the following exercises.

1.1 CLRS [w] 2.1-1.

1.2 CLRS [w] 2.1-2.

1.3 CLRS 2.2-3.

1.4 CLRS [w] 2.3-1.

1.5 CLRS [BSc] 2.3-4.

1.6 CLRS 2.3-6.

2 Duplicates and Close Neighbours Let $A[0..n-1]$ be an array of integers. Solve the following exercises.

2.1 [w] A *duplicate* in A is a pair of entries i and j such that $A[i] = A[j]$. Give an algorithm that determines if there is a duplicate in A in $\Theta(n^2)$ time.

2.2 Give an algorithm that determines if there is a duplicate in A in $\Theta(n \log n)$ time. *Hint*: use merge sort.

2.3 A *closest pair* in A is a pair of entries i and j such that $|A[i] - A[j]|$ is minimal among all the pairs of entries. Give an algorithm that finds a closest pair in A in $\Theta(n \log n)$ time.

3 [BEng†] Implementation of Binary Search Implement the binary search algorithm.

4 Implementation and Correctness of Merge Sort Solve the following exercises.

4.1 [†] Implement the merge algorithm.

4.2 [†] Implement the merge sort algorithm.

4.3 [BSc] Show that merge sort sorts all tables correctly. *Hint*: use induction.

5 2Sum and 3Sum Let $A[0..n-1]$ be an array of integers (positive and negative). The array A has a *2-sum* if there exist two entries i and j such that $A[i] + A[j] = 0$. Similarly, A has a *3-sum* if there exists three entries i , j and k such that $A[i] + A[j] + A[k] = 0$. Solve the following exercises.

5.1 [w] Give an algorithm that determines if A has a 2-sum in $\Theta(n^2)$ time.

5.2 Give an algorithm that determines if A has a 2-sum in $\Theta(n \log n)$ time. *Hint*: use binary search.

5.3 [w] Give an algorithm that determines if A has a 3-sum in $\Theta(n^3)$ time.

5.4 Give an algorithm that determines if A has a 3-sum in $\Theta(n^2 \log n)$ time. *Hint*: use binary search.

5.5 [**] Give an algorithm that determines if A has a 3-sum in $\Theta(n^2)$ time.

6 Selection, Partition, and Quick Sort Let $A[0..n-1]$ be an array of distinct integers. The integer with rank k in A is the k th smallest integer among the integers in A . The median of A is the integer in A with rank $\lfloor (n-1)/2 \rfloor$. Solve the following exercises.

6.1 Give an algorithm that given a k finds the integer with rank k in A in $\Theta(n \log n)$ time.

A partition of A is a separation of A into two arrays A_{low} and A_{high} such that A_{low} contains all integers from A that are smaller than or equal to the median of A and A_{high} contains all the integers from A that are larger than the median of A . Assume in the following that you are given a linear time algorithm to determine the median of an array.

6.2 Give an algorithm to compute a partition of A in $\Theta(n)$ time.

6.3 [*] Give an algorithm to sort A in $\Theta(n \log n)$ time using recursive partition.

6.4 [**] Give an algorithm that given a k finds the integer with rank k in A in $\Theta(n)$ time.