Weekplan: Binary Search Trees
The 02105+02326 DTU Algorithms Team

Reading

Exercises

1 Simulation and Properties
1.1 Which of the following trees are binary search trees?

```
(a) 16
    12 20
    2 11 18 21
  8

(b) 9
    7 18
    2 8 13 17
  50

(c) 16
    17 10
    2 15 13 11
  10
```

1.2 Where are the elements with respectively the smallest and largest key located in a binary search tree?

1.3 CLRS 12.1-1.

1.4 Specify the pre-order, in-order og post-order sequence of keys for the tree in (b)

1.5 CLRS 12.1-2.

1.6 CLRS 12.1-3. Write pseudo code for the algorithm.

1.7 CLRS 12.2-1.

1.8 [BSc] CLRS 12.2-5. *Hint:* prove by contradiction.

2 Leaves and Heights
Let $T$ be a binary tree with $n$ nodes and root $v$.

2.1 Give a recursive algorithm that given $v$ computes the number of leafs in $T$. Write pseudo code for your solution.

2.2 Give a recursive algorithm that given $v$ computes the height of $T$. Write pseudo code for your solution.

2.3 [*] Implement your solution to compute the height.

3 More Recursion on Trees
Solve exercise 4 in the exam set from 2011.

4 Traversal of Binary Search Trees
4.1 Give an algorithm that given a binary search tree $T$ with a key in each node, determines if $T$ satisfies the binary search tree property.

4.2 Give an algorithm that given a binary search tree $T$ constructs a reversed binary search tree $T^R$. $T^R$ should be a binary search tree with the same keys as $T$. For each node $v$ in $T^R$ the nodes in the left subtree must be $\geq v$ and the keys in the right subtree must be $\leq v$.

4.3 [*] Give an algorithm that given two binary search trees $T_1$ and $T_2$ constructs a single binary search tree with all the elements from both $T_1$ and $T_2$. 


5  Perfectly Balanced Binary Search Trees  Let $A$ be a sorted array of $n = 2^{h+1} − 1$ distinct numbers. Give a sequence of insertions of the numbers in $A$ into a binary search tree $T$ such that $T$ becomes a complete binary search tree of height $h$.

6  Pre-Order Traversal  [†] Implement a recursive algorithm for pre-order traversal of a binary tree.

7  Even More Recursion on Trees  Solve exercise 4 in the exam set from 2010.