Weekplan: Searching and Sorting

The 02105+02326 DTU Algorithms Team

Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Chapter 2.

Exercises

1  Run by Hand and Properties Solve the following exercises.
   1.1 CLRS [w] 2.1-1.
   1.2 CLRS [w] 2.1-2.
   1.3 CLRS 2.2-3.
   1.4 CLRS [w] 2.3-1.
   1.5 CLRS [BSc] 2.3-4.
   1.6 CLRS 2.3-6.

2  Duplicates and Close Neighbours Let $A[0..n-1]$ be an array of integers. Solve the following exercises.
   2.1 [w] A duplicate in $A$ is a pair of entries $i$ and $j$ such that $A[i] = A[j]$. Give an algorithm that determines if there is a duplicate in $A$ in $\Theta(n^2)$ time.
   2.2 Give an algorithm that determines if there is a duplicate in $A$ in $\Theta(n \log n)$ time. Hint: use merge sort.
   2.3 A closest pair in $A$ is a pair of entries $i$ and $j$ such that $|A[i] - A[j]|$ is minimal among all the pairs of entries. Give an algorithm that finds a closest pair in $A$ in $\Theta(n \log n)$ time.

3  [BEng†] Implementation of Binary Search Implement the binary search algorithm.

4  Implementation and Correctness of Merge Sort Solve the following exercises.
   4.1 [†] Implement the merge algorithm.
   4.2 [†] Implement the merge sort algorithm.
   4.3 [BSc] Show that merge sort sorts all tables correctly. Hint: use induction.

5  2Sum and 3Sum Let $A[0..n-1]$ be an array of integers (positive and negative). The array $A$ has a 2-sum if there exist two entries $i$ and $j$ such that $A[i] + A[j] = 0$. Similarly, $A$ has a 3-sum if there exists three entries $i$, $j$ and $k$ such that $A[i] + A[j] + A[k] = 0$. Solve the following exercises.
   5.1 [w] Give an algorithm that determines if $A$ has a 2-sum in $\Theta(n^2)$ time.
   5.2 Give an algorithm that determines if $A$ has a 2-sum in $\Theta(n \log n)$ time. Hint: use binary search.
   5.3 [w] Give an algorithm that determines if $A$ has a 3-sum in $\Theta(n^3)$ time.
   5.4 Give an algorithm that determines if $A$ has a 3-sum in $\Theta(n^2 \log n)$ time. Hint: use binary search.
   5.5 [***] Give an algorithm that determines if $A$ has a 3-sum in $\Theta(n^2)$ time.
6 Selection, Partition, and Quick Sort  Let $A[0..n-1]$ be an array of distinct integers. The integer with rank $k$ in $A$ is the $k$th largest integer among the integers in $A$. The median of $A$ is the integer in $A$ with rank $\lceil (n-1)/2 \rceil$. Solve the following exercises.

6.1 Give an algorithm that given a $k$ finds the integer with rank $k$ in $A$ in $\Theta(n \log n)$ time.

6.2 Give an algorithm to compute a partition of $A$ in $\Theta(n)$ time.

6.3 [*] Give an algorithm to sort $A$ in $\Theta(n \log n)$ time using recursive partition.

6.4 [**] Give an algorithm that given a $k$ finds the integer with rank $k$ in $A$ in $\Theta(n)$ time.