Weekplan: Binary Search Trees

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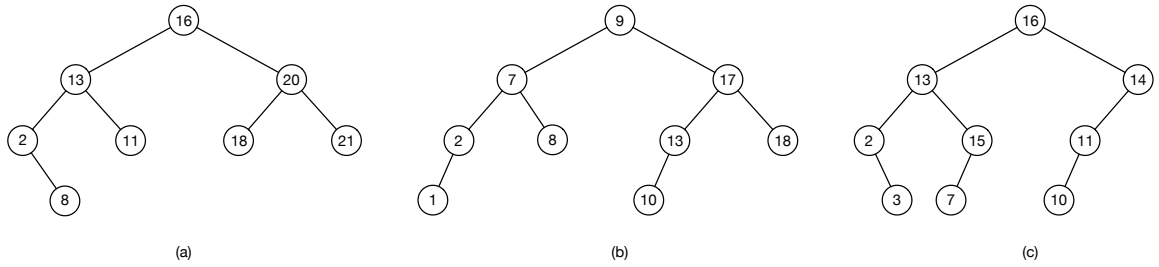
Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Chapter 12 excluding 12.4.

Exercises

1 Simulation and Properties

1.1 [w] Which of the following trees are binary search trees?



1.2 [w] Where are the elements with respectively the smallest and largest key located in a binary search tree?

1.3 [w] CLRS 12.1-1.

1.4 [w] Specify the pre-order, in-order og post-order sequence of keys for the tree in (b)

1.5 CLRS 12.1-2.

1.6 CLRS 12.1-3. Write pseudo code for the algorithm.

1.7 CLRS 12.2-1.

1.8 [BSc] CLRS 12.2-5. *Hint:* prove by contradiction.

2 Leafs and Heights

Let T be a binary tree with n nodes and root v.

2.1 Give a recursive algorithm that given v computes the number of leafs in T. Write pseudo code for your solution.

2.2 Give a recursive algorithm that given v computes the height of T. Write pseudo code for your solution.

2.3 [†] Implement your solution to compute the height.

3 More Recursion on Trees

Solve exercise 4 in the exam set from 2011.

4 Traversal of Binary Search Trees

4.1 Give an algorithm that given a binary search tree T with a key in each node, determines if T satisfies the binary search tree property.

4.2 Give an algorithm that given a binary search tree T constructs a *reversed binary search tree* T^R. T^R should be a binary search tree with the same keys as T. For each node v in T^R the nodes in the left subtree must be $\geq v$ and the keys in the right subtree must be $\leq v$.

4.3 [$*$] Give an algorithm that given two binary search trees T_1 and T_2 constructs a single binary search tree with all the elements from both T_1 and T_2.

5 Perfectly Balanced Binary Search Trees Let A be a sorted array of $n = 2^{h+1} - 1$ distinct numbers. Give a sequence of insertions of the numbers in A into a binary search tree T such that T becomes a complete binary search tree of height h.

6 Pre-Order Traversal [†] Implement a recursive algorithm for pre-order traversal of a binary tree.

7 Even More Recursion on Trees Solve exercise 4 in the exam set from 2010.