Weekplan: Binary Search Trees
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Reading

Introduction to Algorithms, Cormen, Rivest, Leisersons and Stein (CLRS): Chapter 12 excluding 12.4.

Exercises

1 Simulation and Properties

1.1 \([w]\) Which of the following trees are binary search trees?

![Binary Search Trees Diagram]

1.2 \([w]\) Where are the elements with respectively the smallest and largest key located in a binary search tree?

1.3 \([w]\) CLRS 12.1-1.

1.4 \([w]\) Specify the pre-order, in-order og post-order sequence of keys for the tree in (b)

1.5 CLRS 12.1-2.

1.6 CLRS 12.1-3. Write pseudo code for the algorithm.

1.7 CLRS 12.2-1.

1.8 [BSc] CLRS 12.2-5. Hint: prove by contradiction.

2 Leafs and Heights

Let \(T\) be a binary tree with \(n\) nodes and root \(v\).

2.1 Give a recursive algorithm that given \(v\) computes the number of leafs in \(T\). Write pseudo code for your solution.

2.2 Give a recursive algorithm that given \(v\) computes the height of \(T\). Write pseudo code for your solution.

2.3 [†] Implement your solution to compute the height.

3 More Recursion on Trees

Solve exercise 4 in the exam set from 2011.

4 Traversal of Binary Search Trees

4.1 Give an algorithm that given a binary search tree \(T\) with a key in each node, determines if \(T\) satisfies the binary search tree property.

4.2 Give an algorithm that given a binary search tree \(T\) constructs a reversed binary search tree \(T^R\). \(T^R\) should be a binary search tree with the same keys as \(T\). For each node \(v\) in \(T^R\) the nodes in the left subtree must be \(\geq v\) and the keys in the right subtree must be \(\leq v\).

4.3 [∗] Give an algorithm that given two binary search trees \(T_1\) and \(T_2\) constructs a single binary search tree with all the elements from both \(T_1\) and \(T_2\).
5 Perfectly Balanced Binary Search Trees  Let $A$ be a sorted array of $n = 2^{h+1} - 1$ distinct numbers. Give a sequence of insertions of the numbers in $A$ into a binary search tree $T$ such that $T$ becomes a complete binary search tree of height $h$.

6 Pre-Order Traversal  [†] Implement a recursive algorithm for pre-order traversal of a binary tree.

7 Even More Recursion on Trees  Solve exercise 4 in the exam set from 2010.