Deformable Simplicial Complex

Marek K. Misztal

Informatics and Mathematical Modelling, Technical University of Denmark
mkm@imm.dtu.dk

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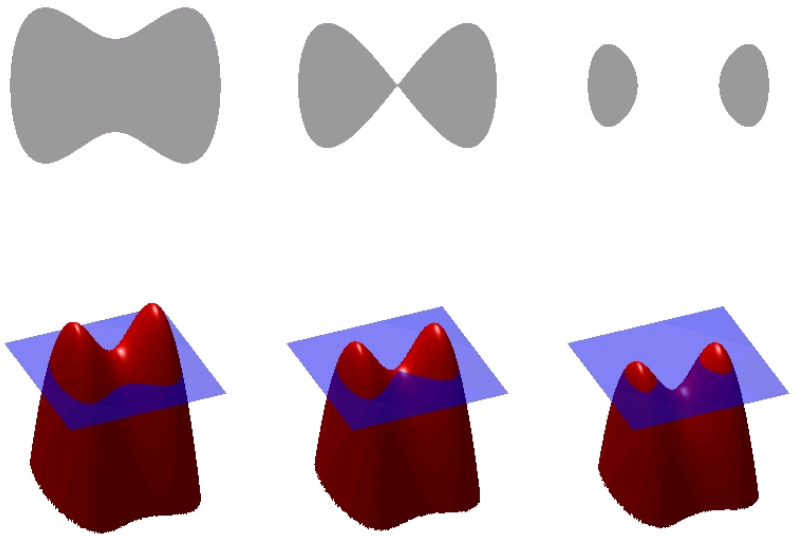
  - represent the n-dimensional interface as the 0-level set of a $(n+1)$-dimensional function $\varphi(x_1, x_2, \ldots, x_{n+1})$,
  - deformation is produced through evolution of the function φ, rather than the interface itself.

Level Set Method



The function is defined on the nodes of a rectangular grid.

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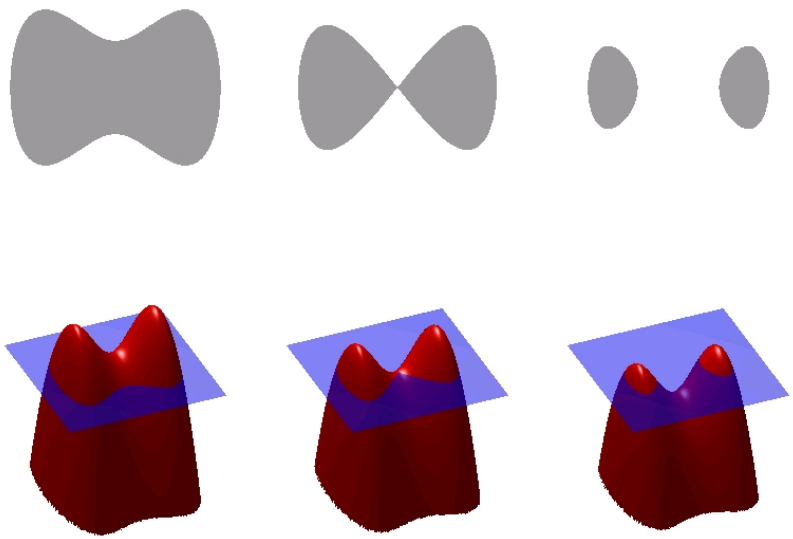

Evolution of the interface due to the velocity field **v** is described by the following PDE (*level set equation*):

$$\frac{\partial \varphi}{\partial t} + \mathbf{u} \cdot \nabla \varphi = 0.$$

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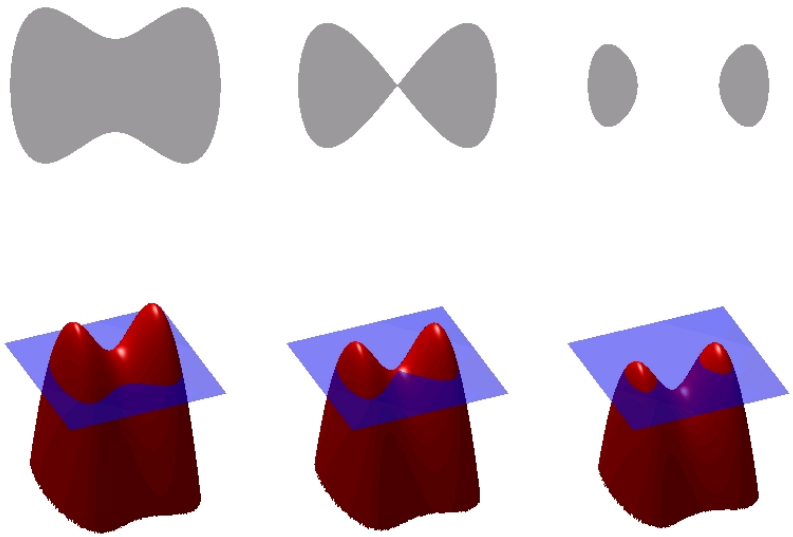

Signed distance function is a popular choice for φ.

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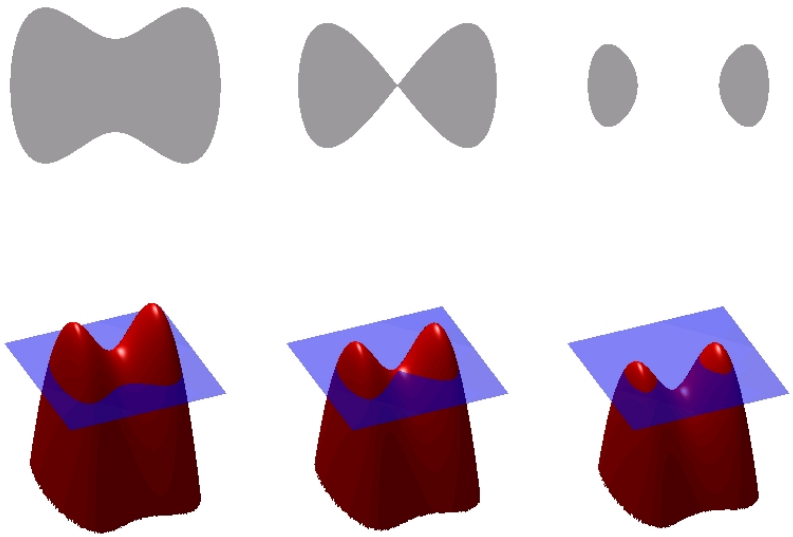

This approach provides trivial and robust topological adaptivity.

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- Extra dimension needed.

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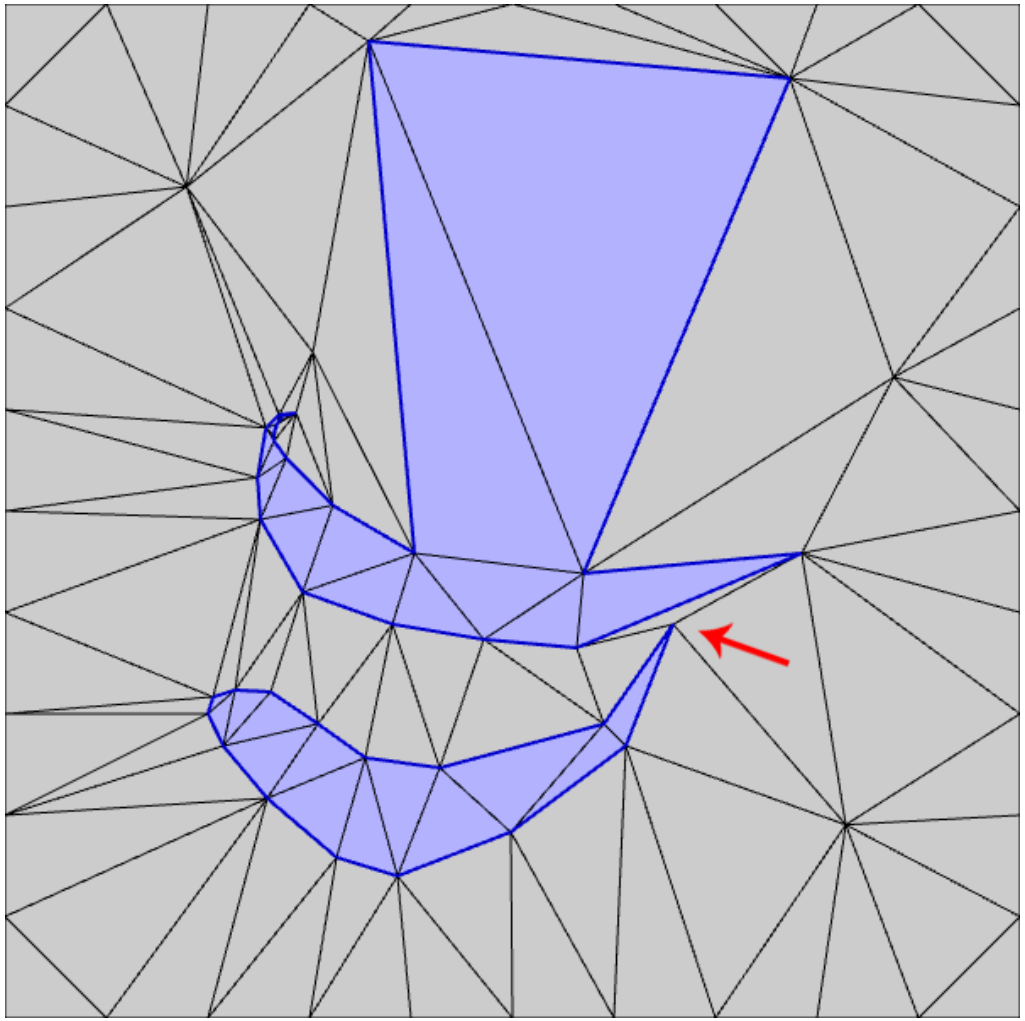
- The interface deformation is performed via displacements of the interface vertices, while keeping the simplicial complex criterion of the underlying triangulation (tetrahedralization) all the time.

2D DSC



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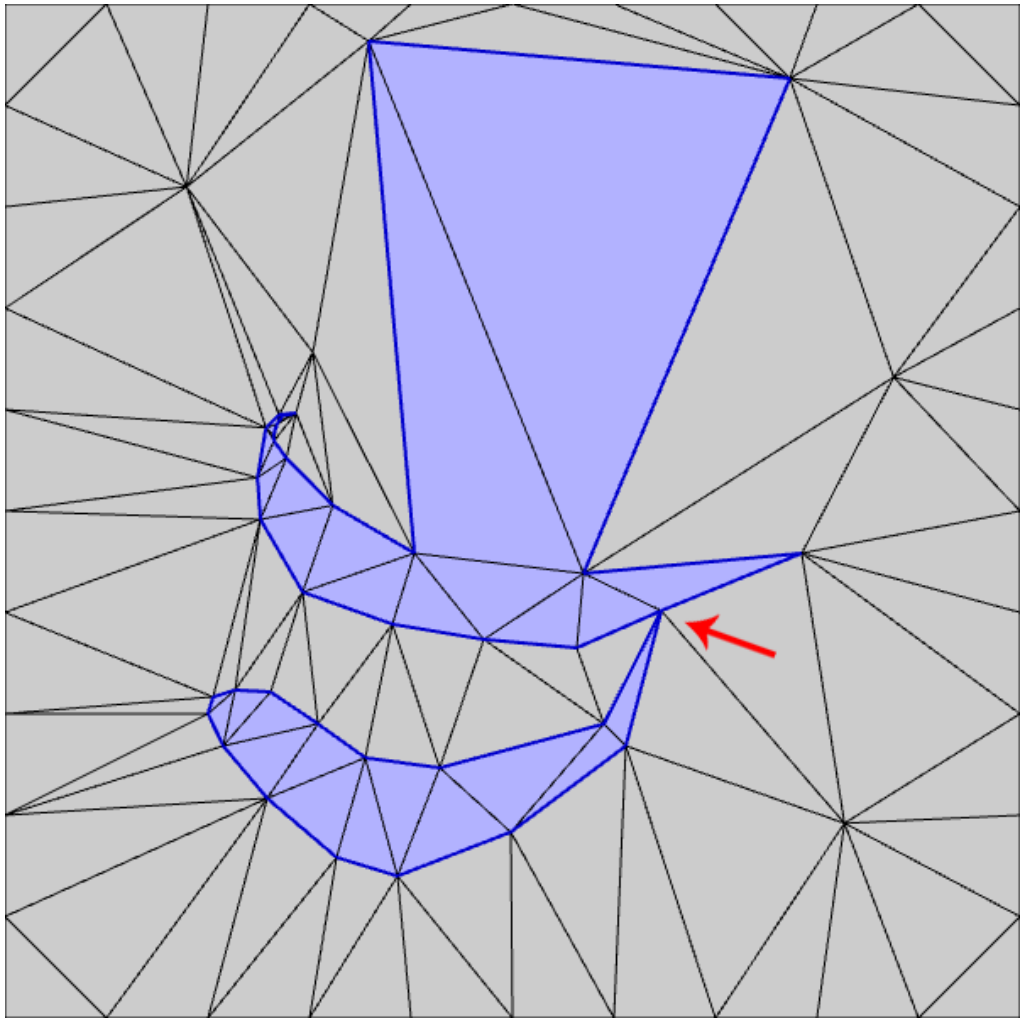

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5. Steps 3 and 4 are performed until all vertices are moved to the new positions.

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- Allows for topology control.

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- **degenerate triangles removal**: *cap* flips, allowing the interface topology to change.

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- Delaunay meshes, useful in the 2D case, tend to contain *slivers* (flat, nearly degenerate tetrahedra) unless special rules regarding to vertex placement apply.
- Laplacian smoothing may produce inverted tetrahedra in 3D despite reasonable performance in 2D.

Quality measure

Instead of using the Delaunay quality measure of a tetrahedron (minimum solid angle) in mesh quality improvement operations, we aimed for a quality measure which would penalize both flat tetrahedra, characterized by near-zero volume despite the fact that the distances between their vertices (edge lengths) can be large.

We have chosen to improve the volume-length ratio of a tetrahedron:

$$Q(\sigma) = 6\sqrt{2}\frac{V(\sigma)}{l_{rms}^3},$$

where $V(t)$ is the oriented volume of a tetrahedron t, and l_{rms} is the average (root-mean-squared) of the lengths of its edges:

$$l_{\mathrm{rms}} = \sqrt{\frac{l_{12}^2 + l_{13}^2 + l_{14}^2 + l_{23}^2 + l_{24}^2 + l_{34}^2}{6}}.$$

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- It contains the following types of operations:
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  - detail control through edge splits and edge collapses;
  - degenerate tetrahedron removal, through relabelling and more aggressive methods;
- In order to optimize the performance, we only perform one pass of improvement for the whole mesh per iteration. In other vertex displacement passes, we only perform improvement for the set of tetrahedra adjacent to the vertices, which have not arrived at their final positions yet.

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- We are using **smart Laplacian smoothing** (where the displacement is only performed if it improves the quality locally) performs much better, however it sometimes fails to improve the mesh quality significantly.
- If that is the case and the minimum quality in the star of a vertex v is lower than 0.1, we perform **optimization-based smoothing** (by L. Freitag et al.), which tries to maximize the minimum quality in the star of v. This is clearly a non-smooth optimization problem.

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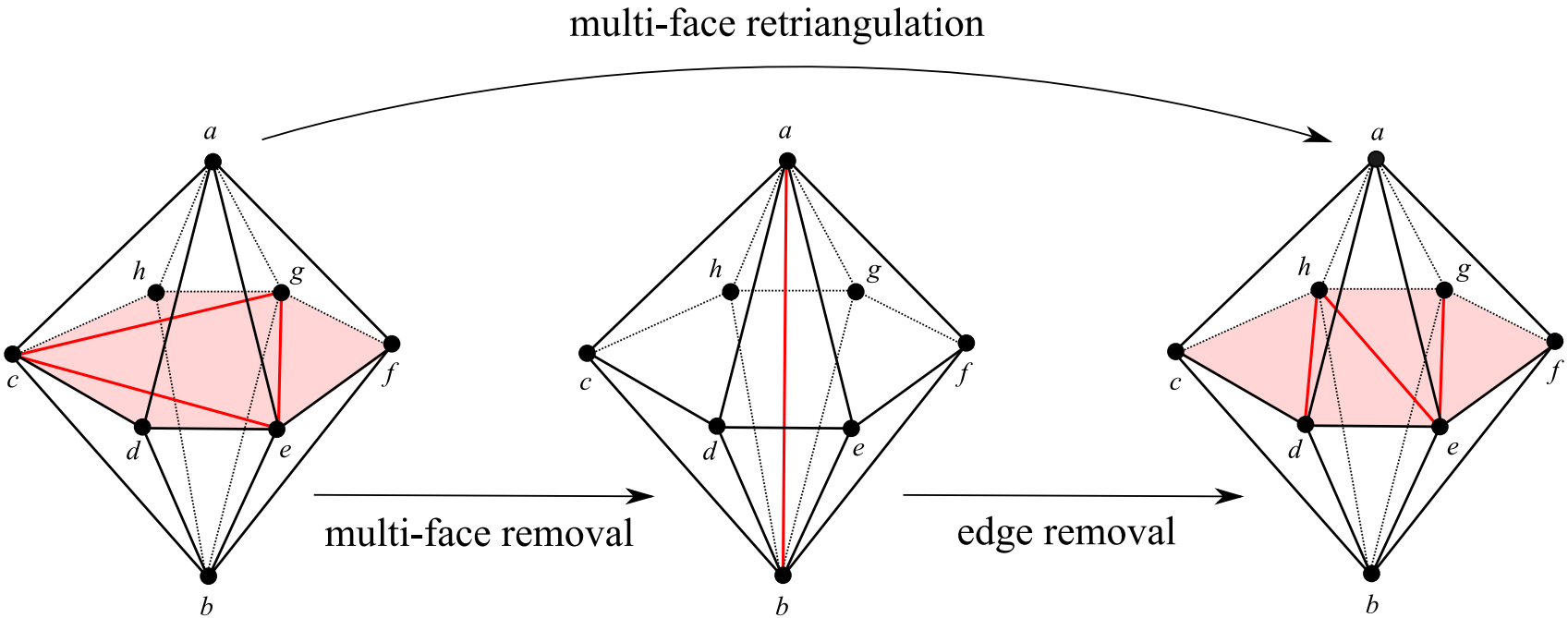
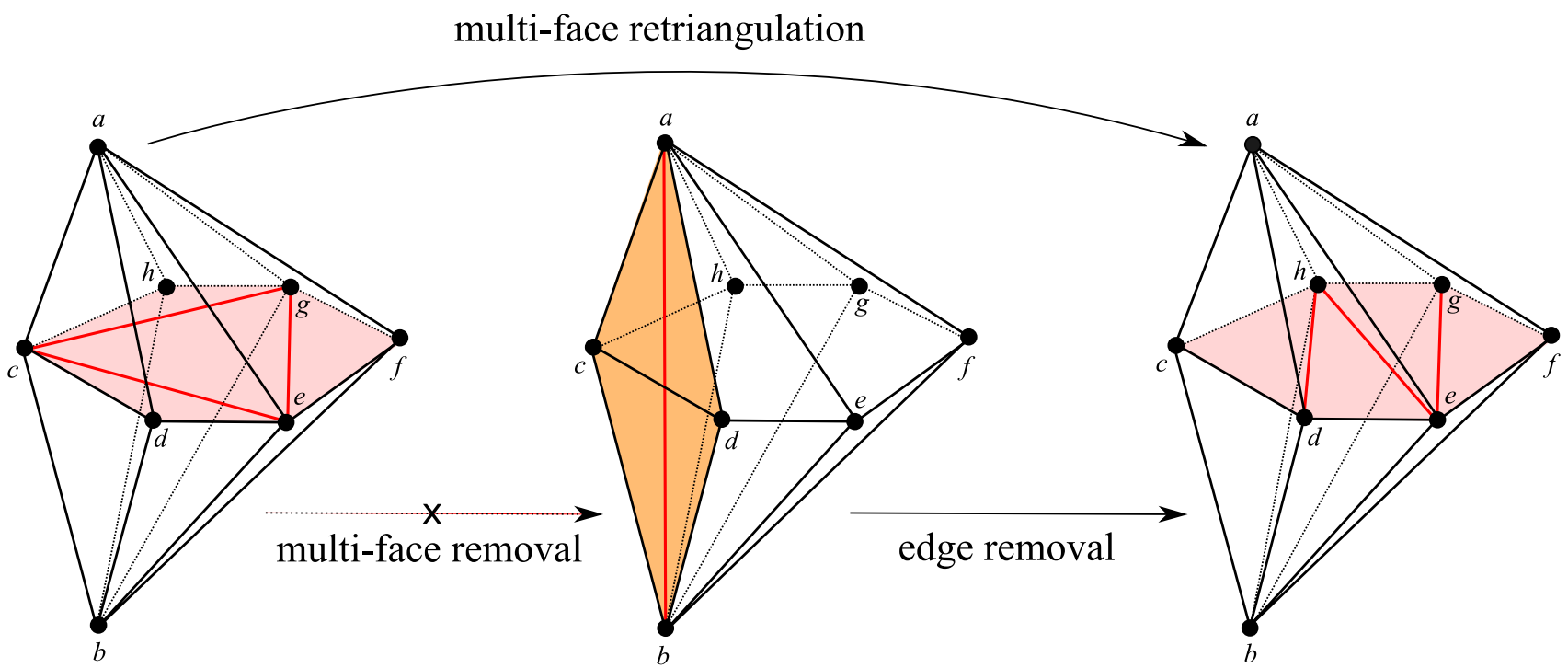
  - multi-face retriangulation.

Topological operations (reconnection)



Multi-face retriangulation



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- Analogously MFRT is also using *Klincsek's algoritm* in order to determine the final triangulation of the polygon "sandwiched" between two apices.
- Multi-face removal requires finding the initial set of tetrahedra to be removed. This is also done using dynamic programming.

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  2. Attempt to remove each face using multi-face retriangulation and multi-face removal. Either of this operation is only performed if it increases the minimum quality over the set of tetrahedra affected by the operation.

Detail control

The purpose of detail control is to ensure that the embedding mesh has enough Steiner vertices, so that it is flexible enough to accomodate changes to the interface and, on the other hand, to make sure that the number of Steiner vertices does not increase too much, in order to keep the performance efficient.

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- **edge split**: for long edges connecting two interface vertices, or an interface vertex with a boundary vertex;
- **edge collapse**: for short edges, not connected to either the interface or the boundary of the embedding mesh, as long as it increases the minimum quality locally or does not decrease it below a certain threshold value.

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- *tetrahedron re-labelling*: switching the label of a tetrahedron from *outside* to *inside* (or the other way round) if it is “squeezed” between two parts of the interface.
- *degeneracy removal*: removing nearly flat tetrahedra by “flattening” them and replacing with 4, 3 or 2 faces. Similarily, we remove degenerate (nearly colinear) faces and degenerate (very short) edges.

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  - *edge splits*: for edges longer than a given threshold.

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- The 3D DSC implementation relies on our own C++ implementation of the Incidence Simplicial data structure for 3D simplicial complexes by de Floriani et al.
- In its current form, the 3D DSC algorithm requires a triangle mesh as an input. This triangle mesh is then normalized and tetrahedralized on the inside and on the outside using TetGen (the outside mesh is bounded by a sparsly subdivided $(-1;-1;-1) \times (1;1;1)$ box).

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- **Offsetting:** performed through face offsetting, rather than displacing the vertices by a constant distance in the approximate normal direction.

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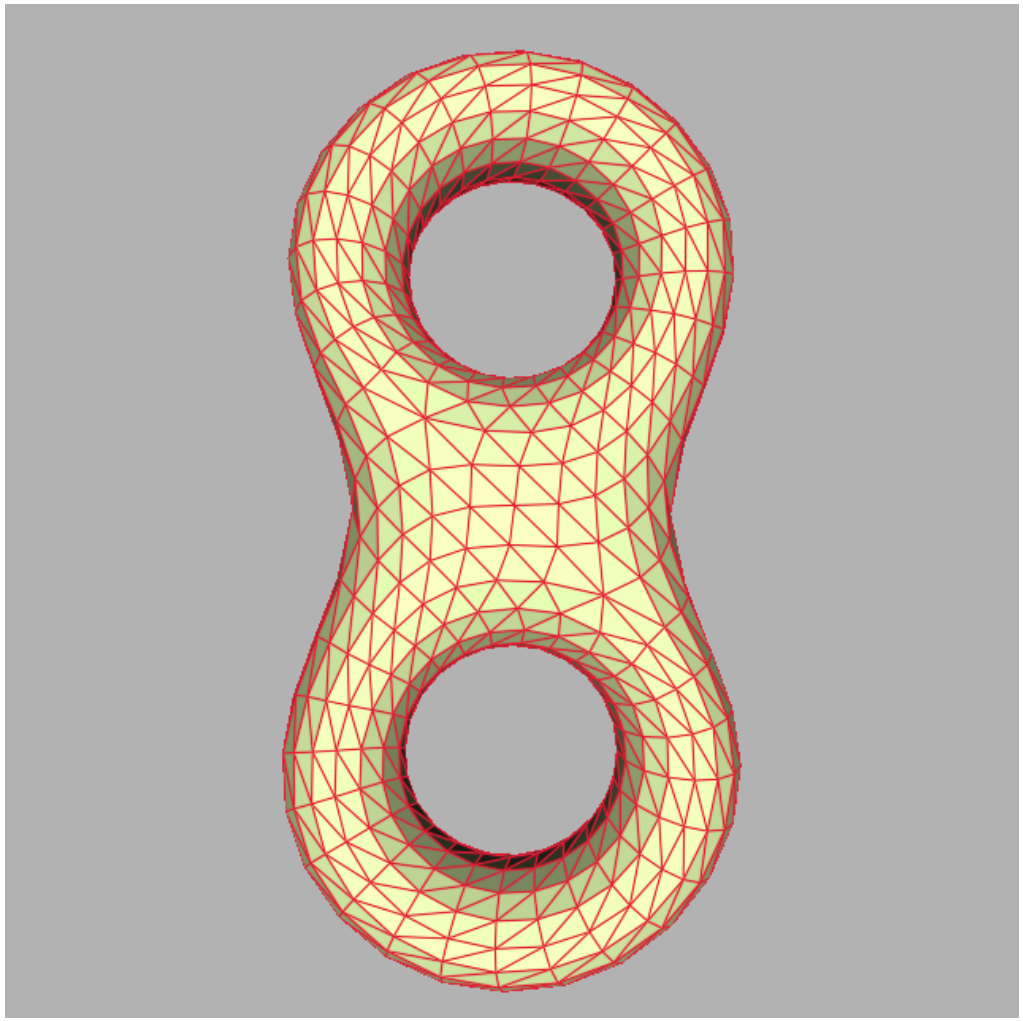

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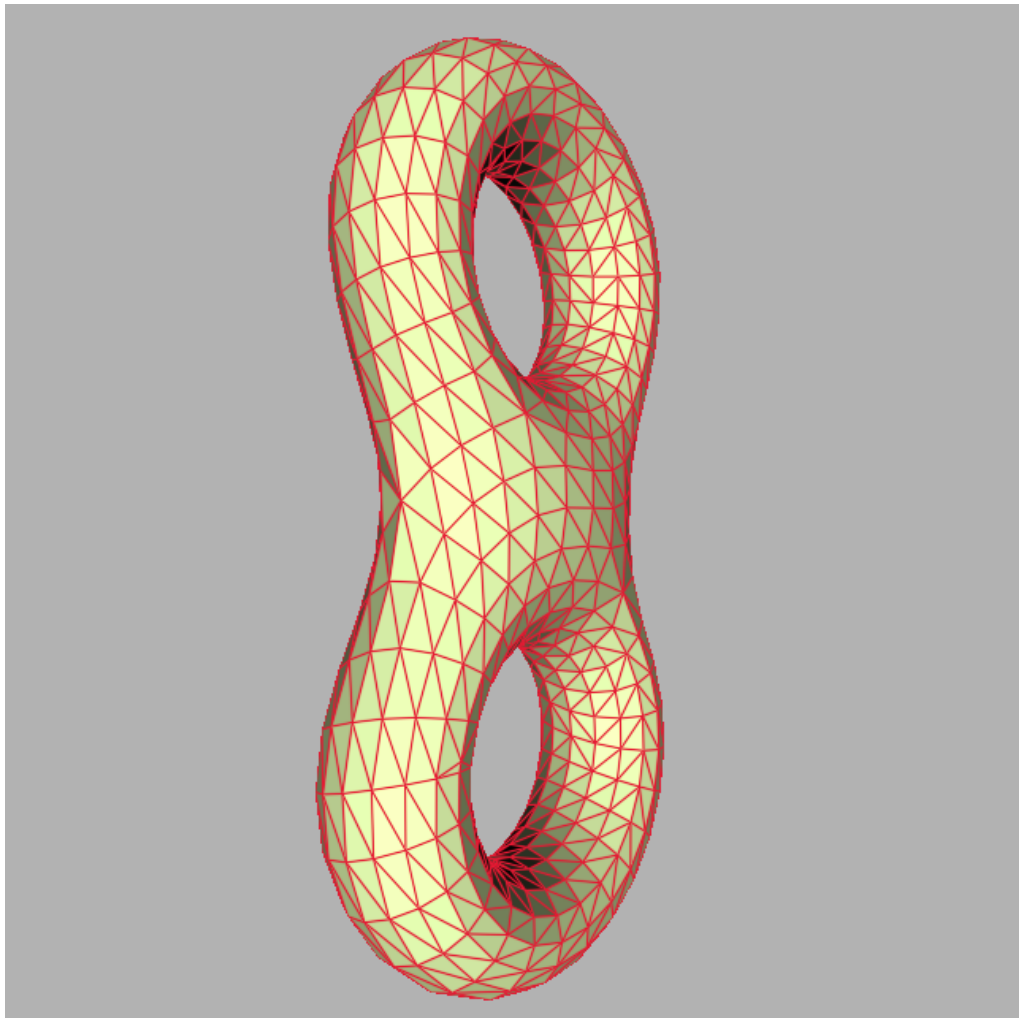

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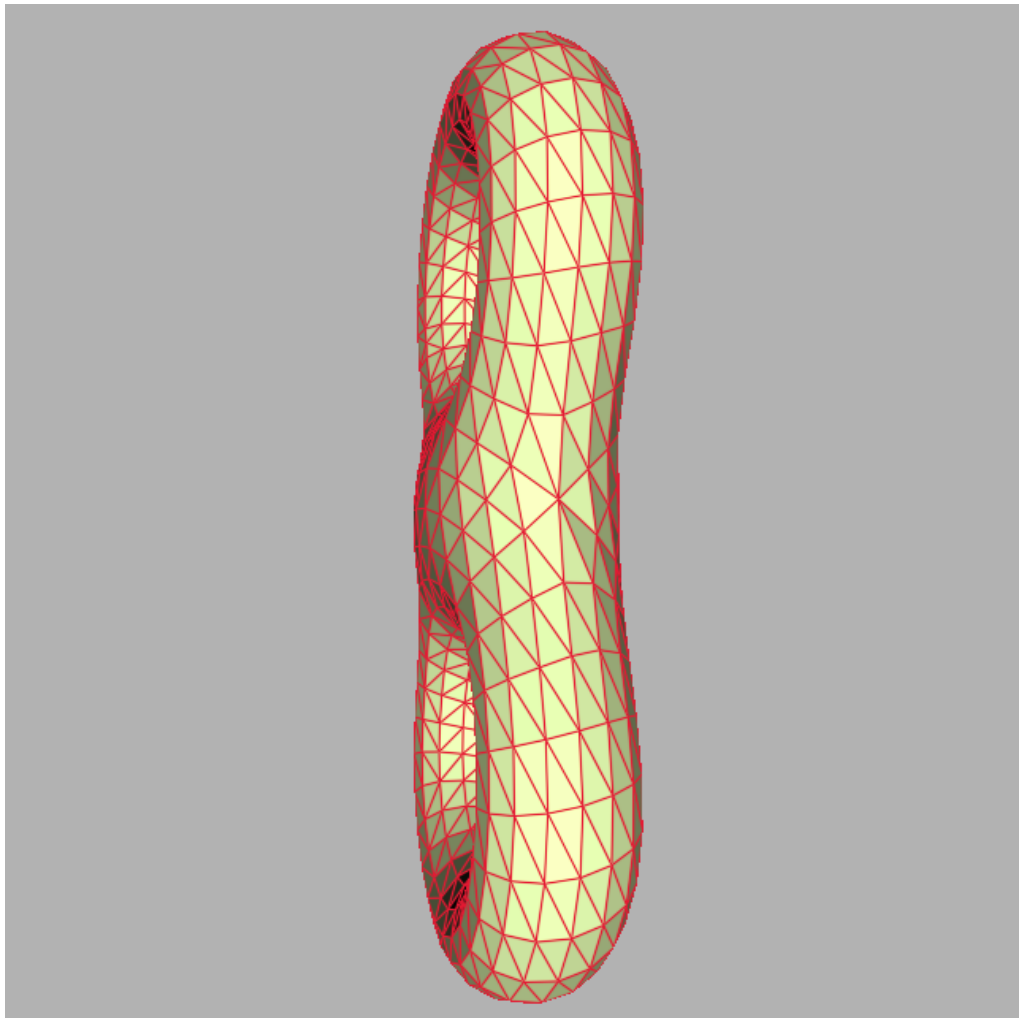

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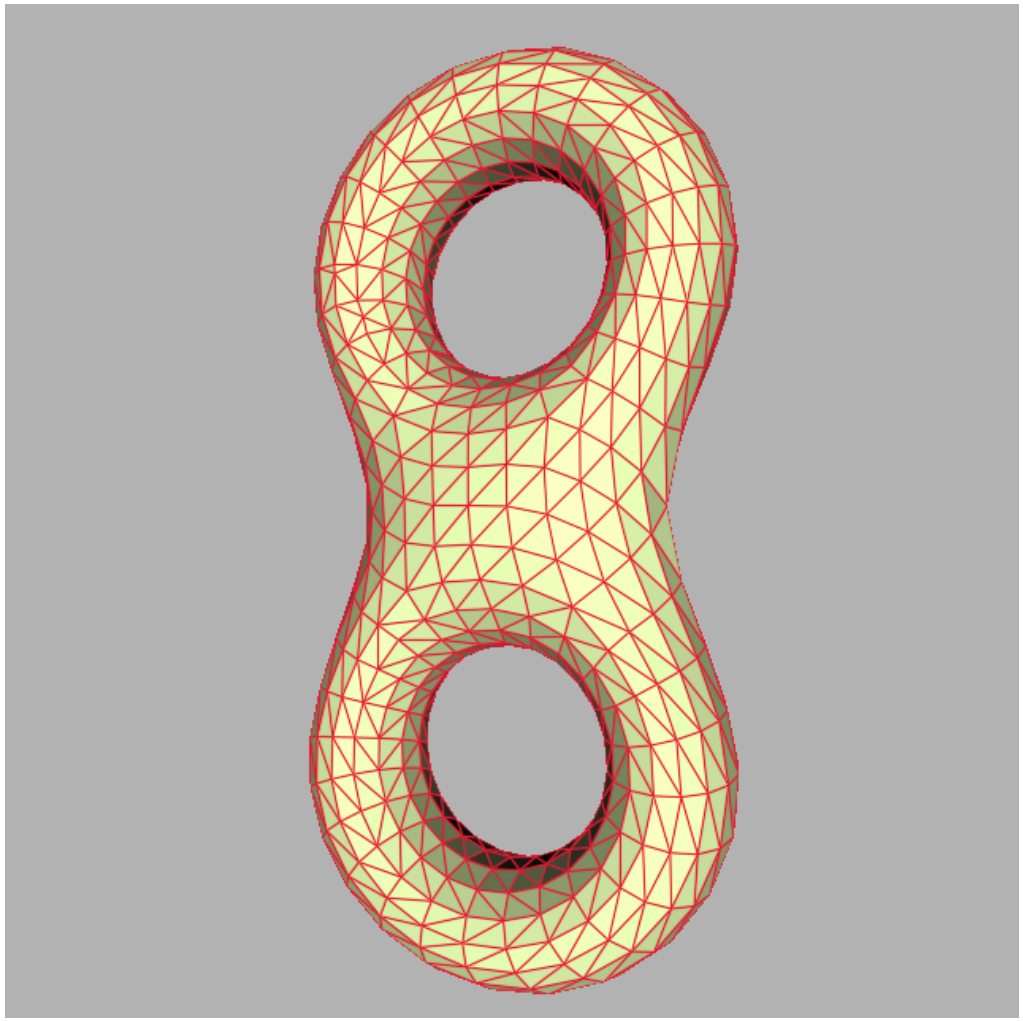

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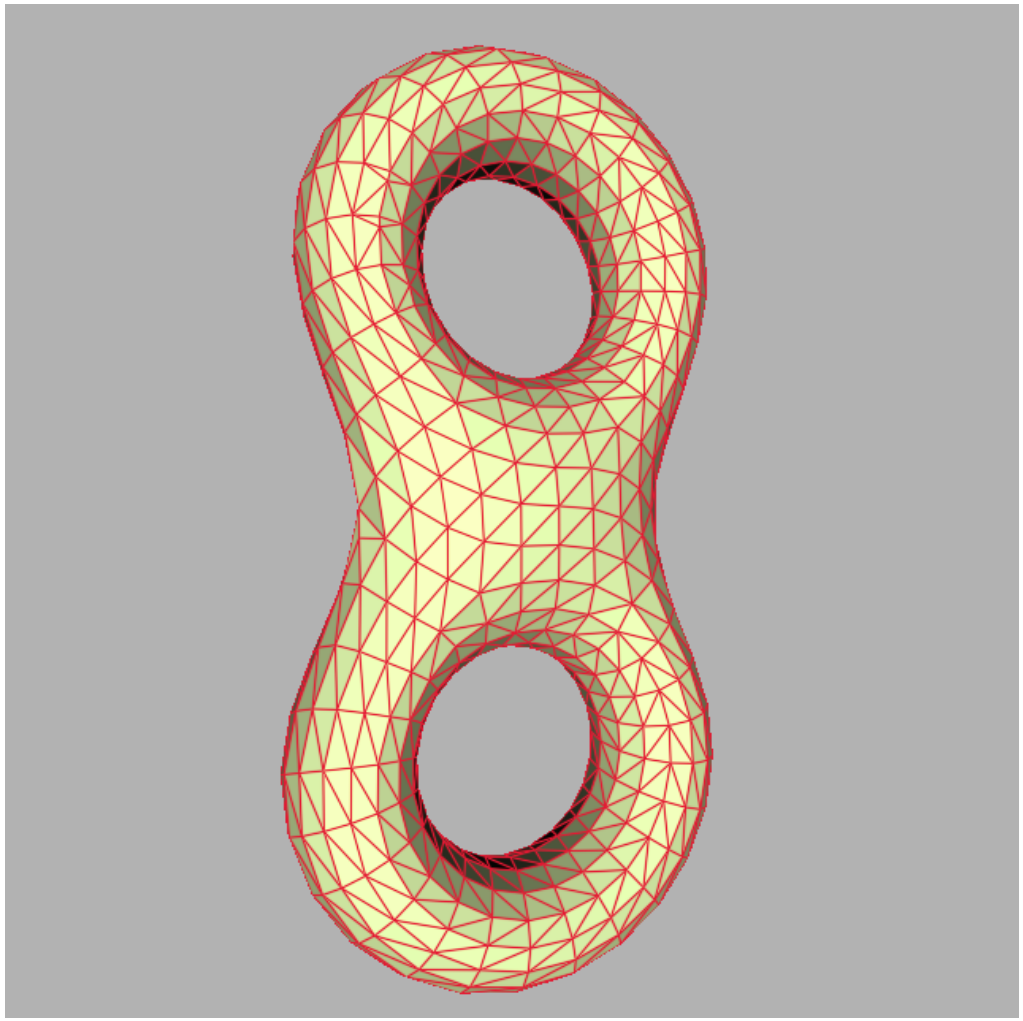

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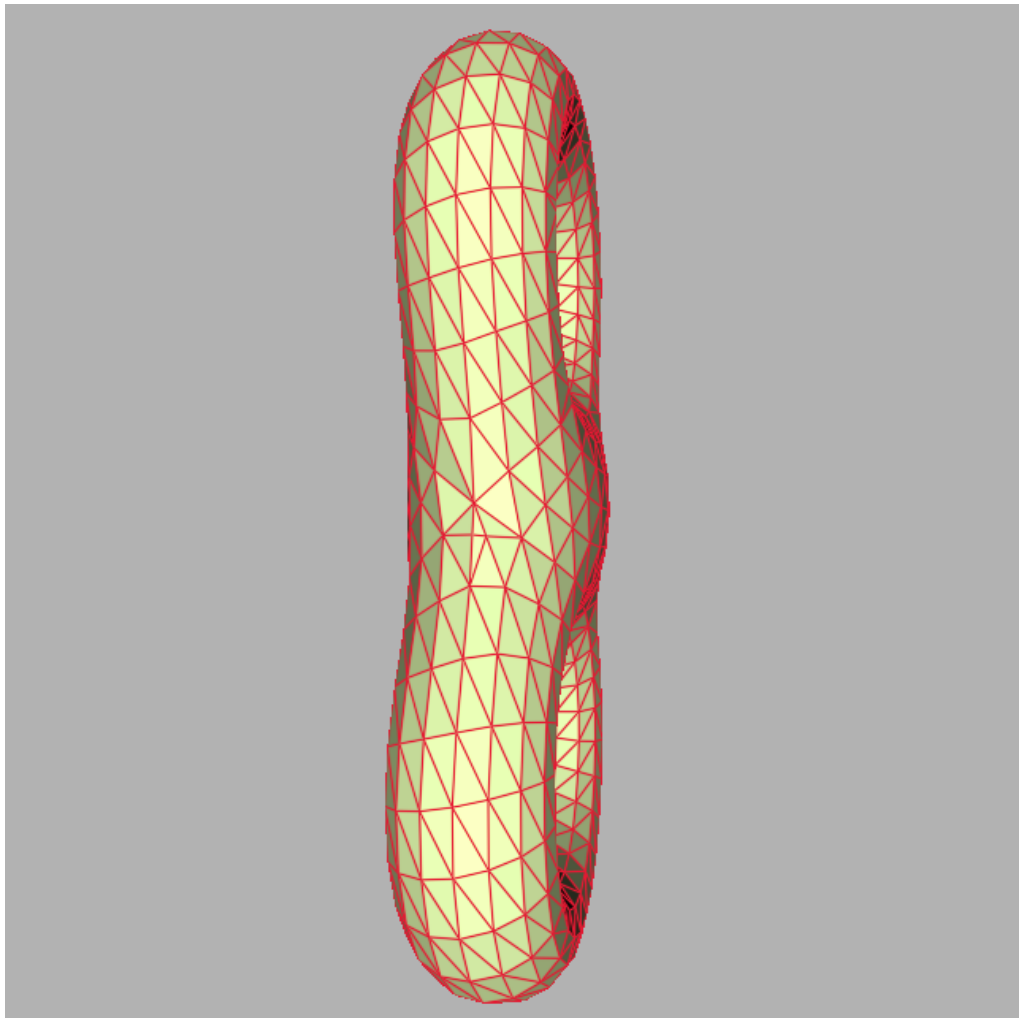

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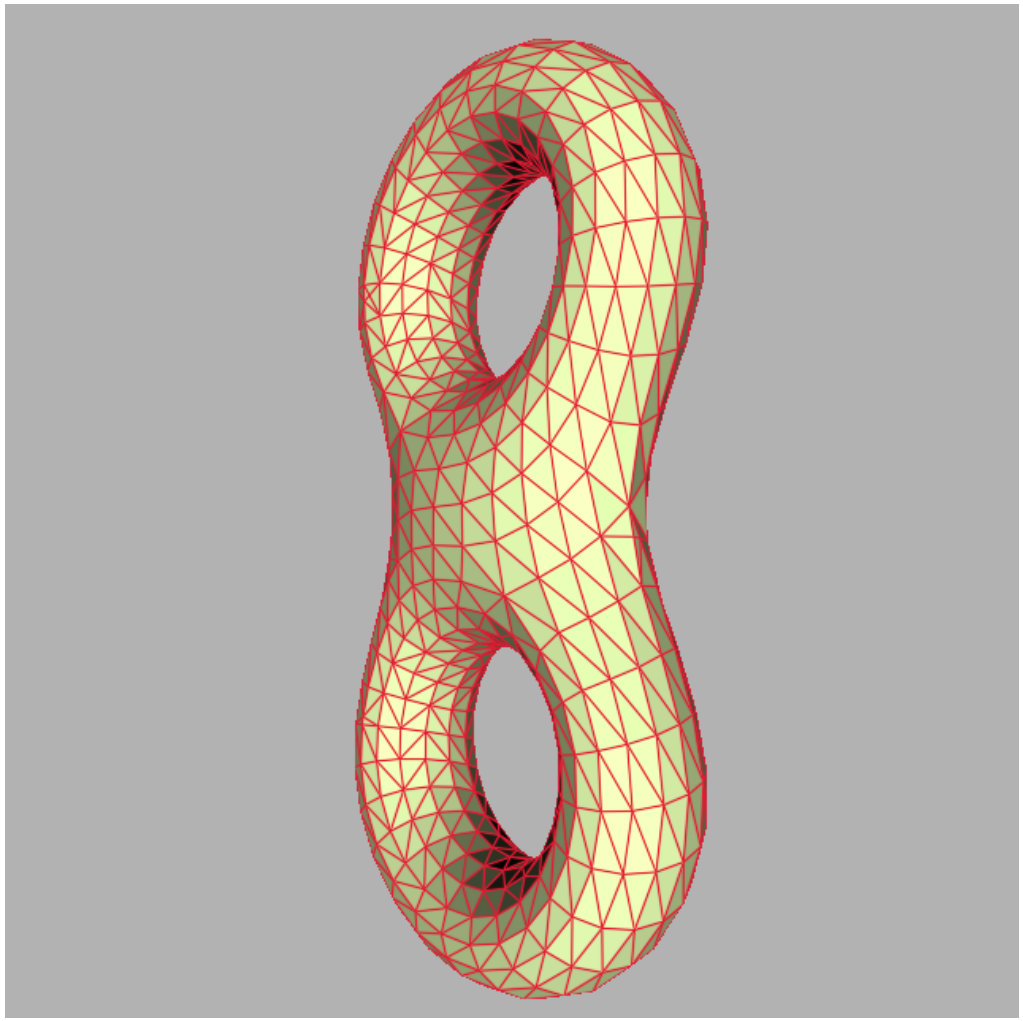

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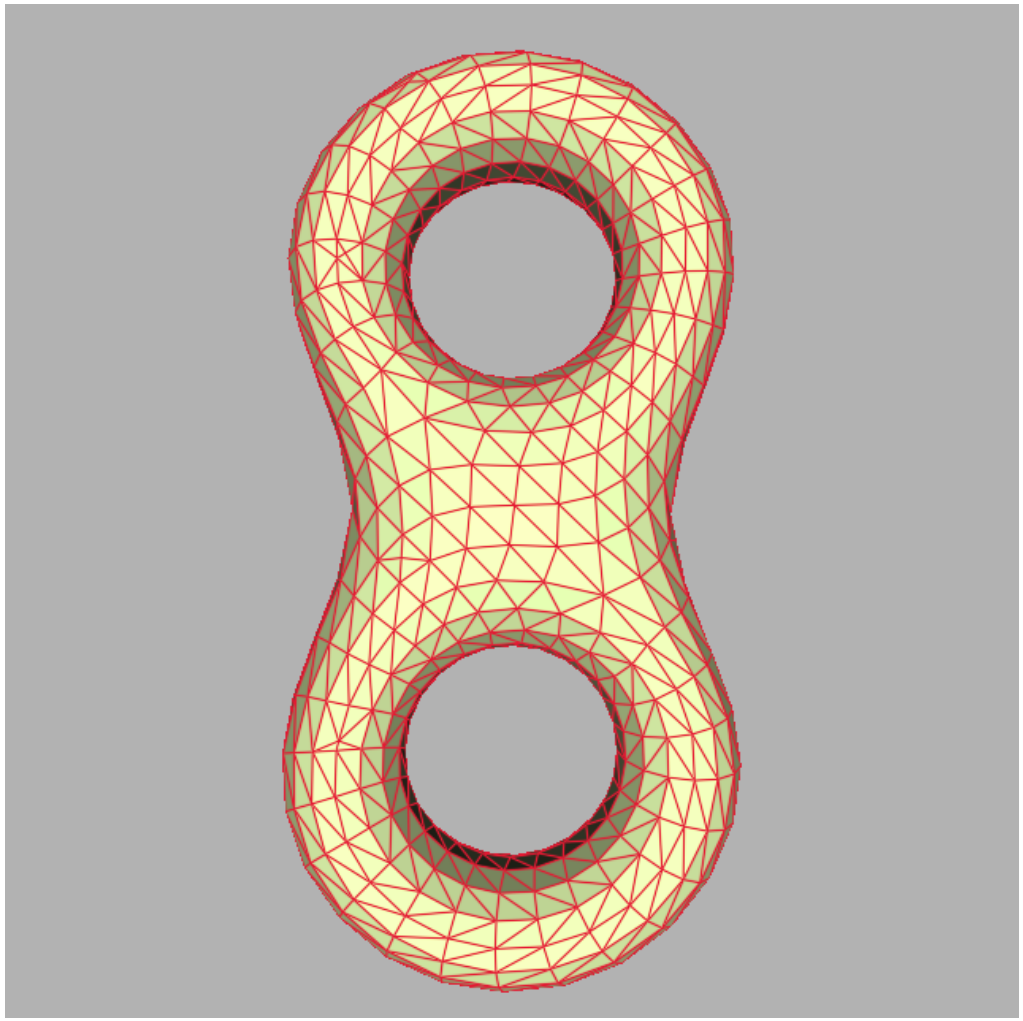
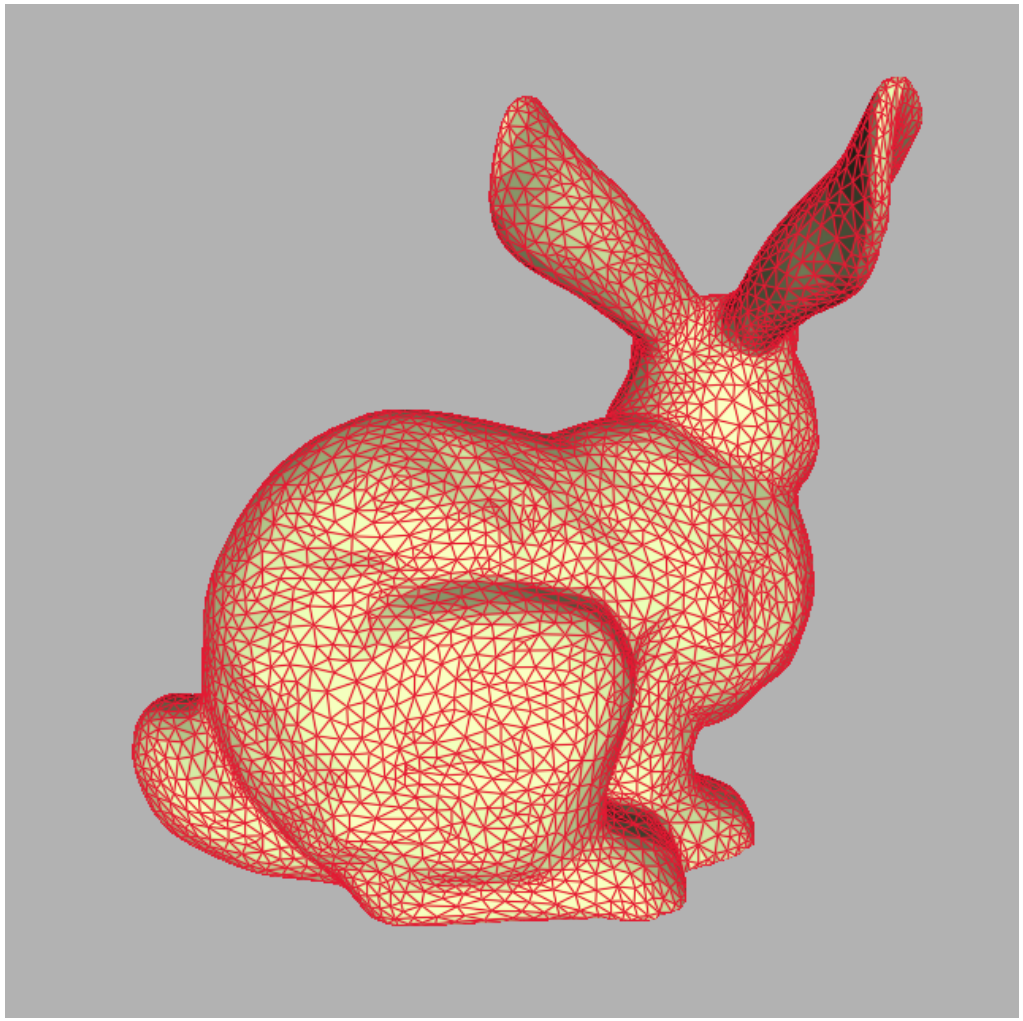


Mean curvature flow



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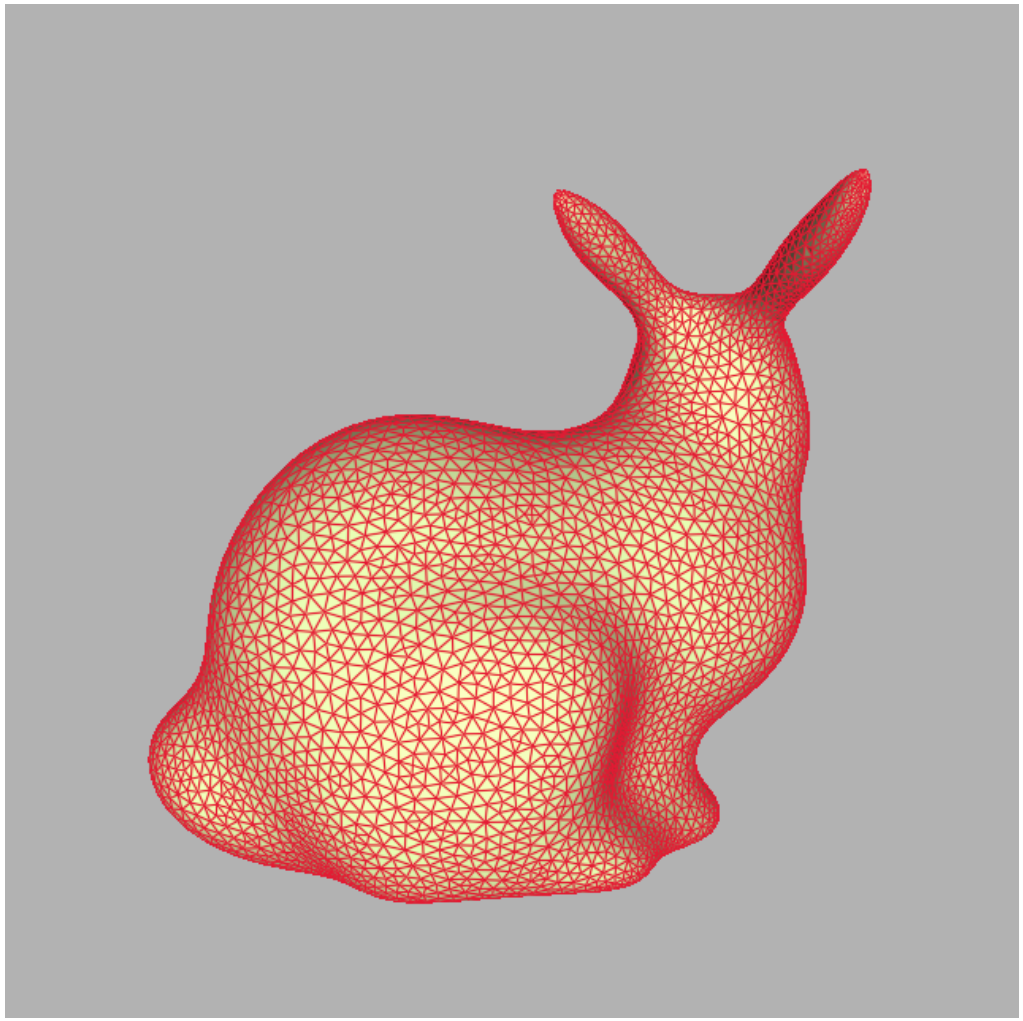

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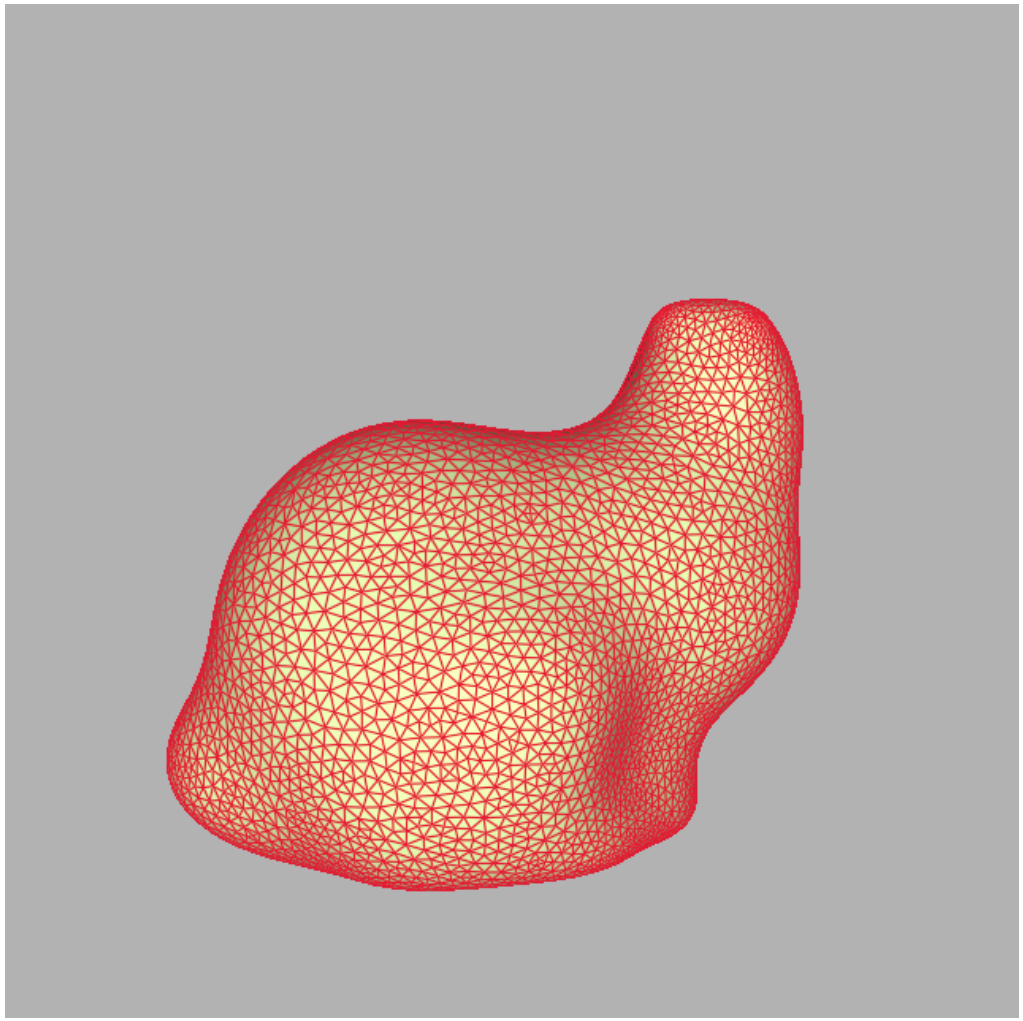
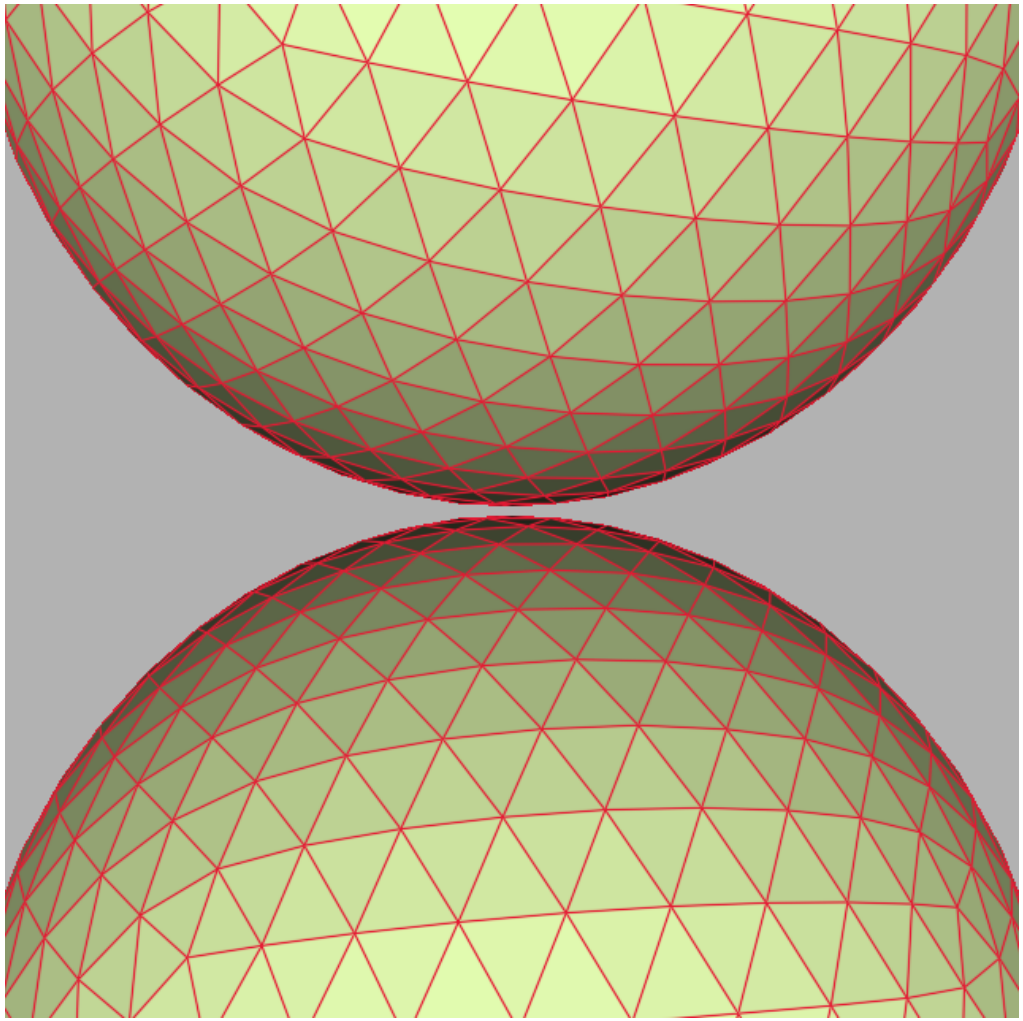


Offsetting



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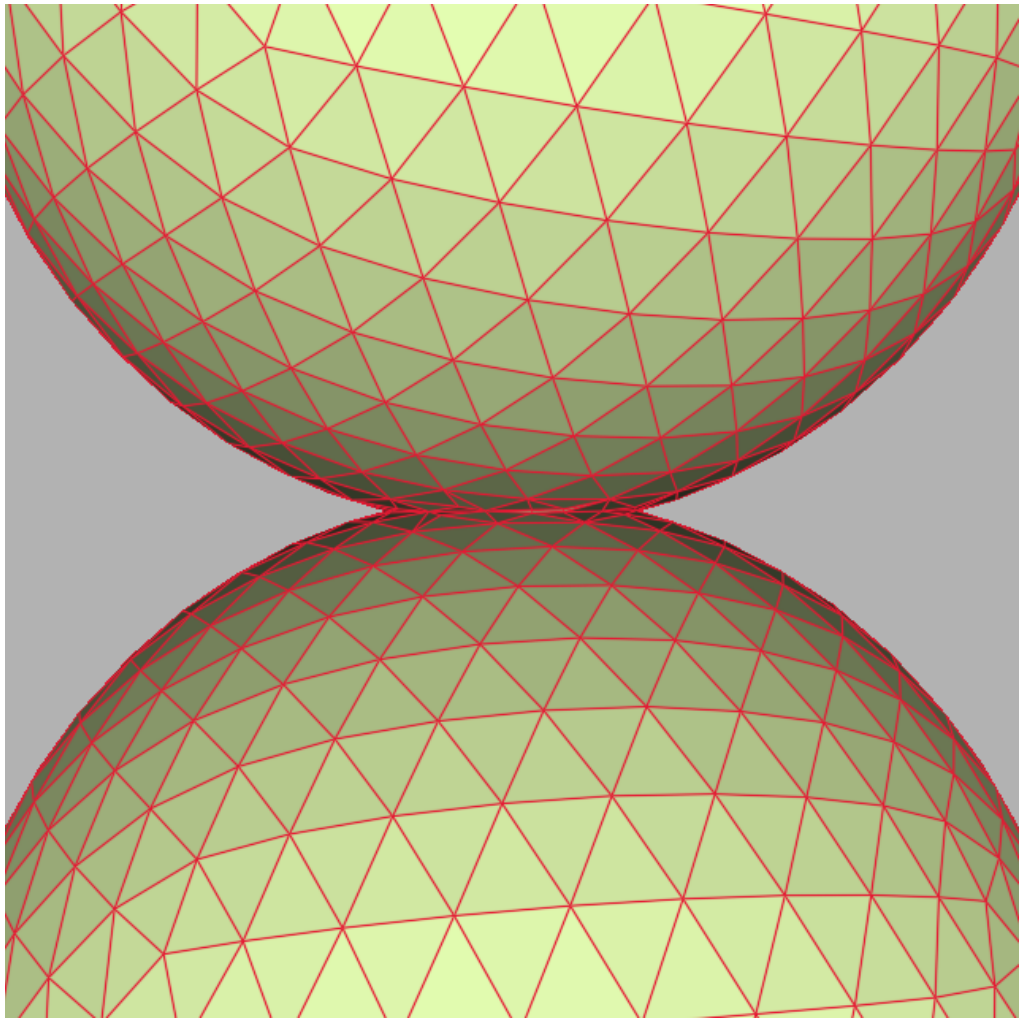

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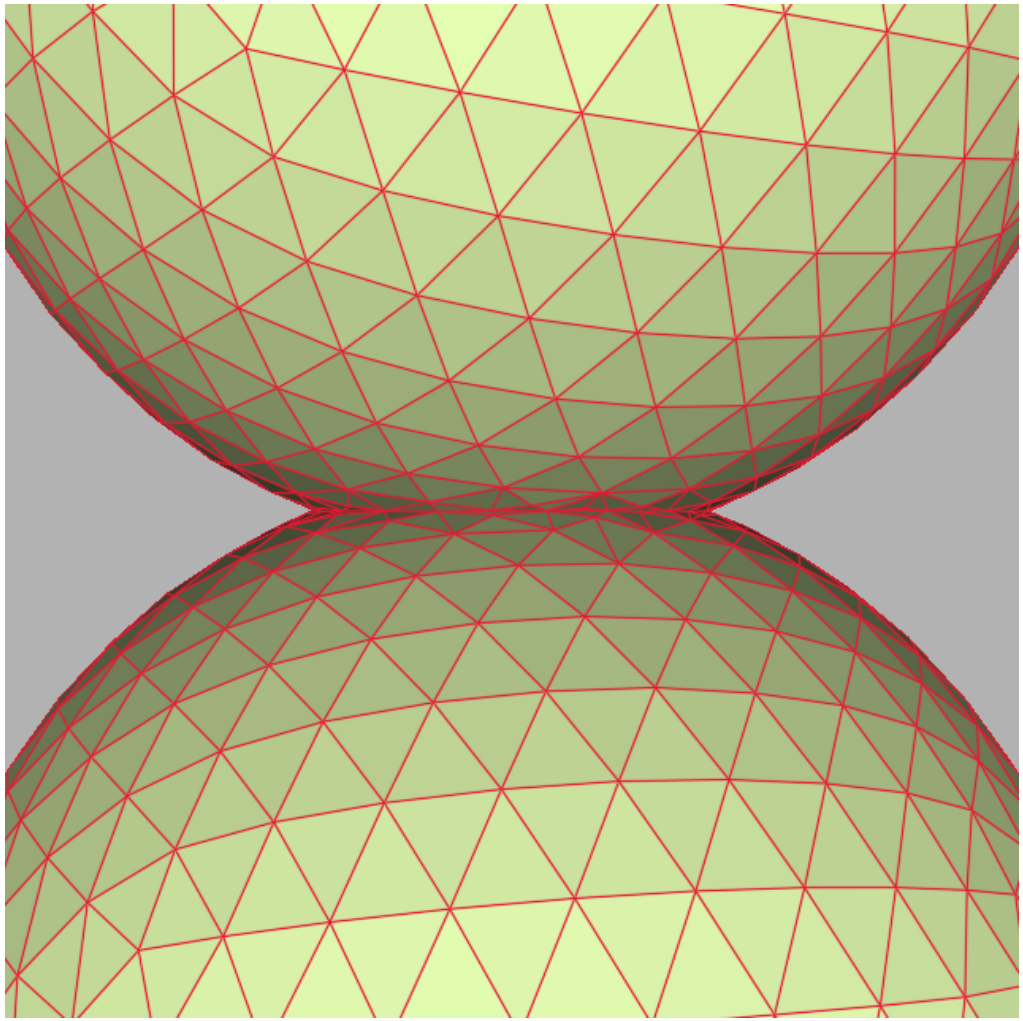

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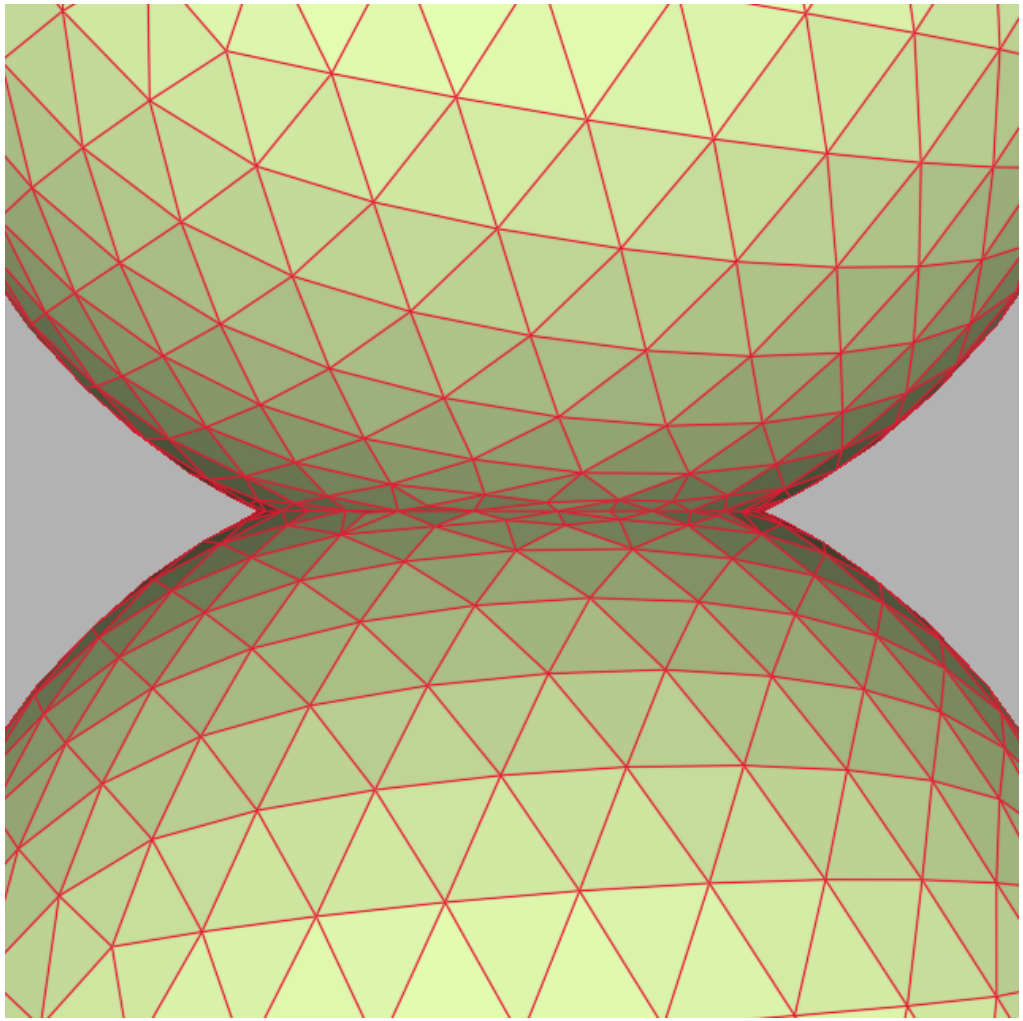

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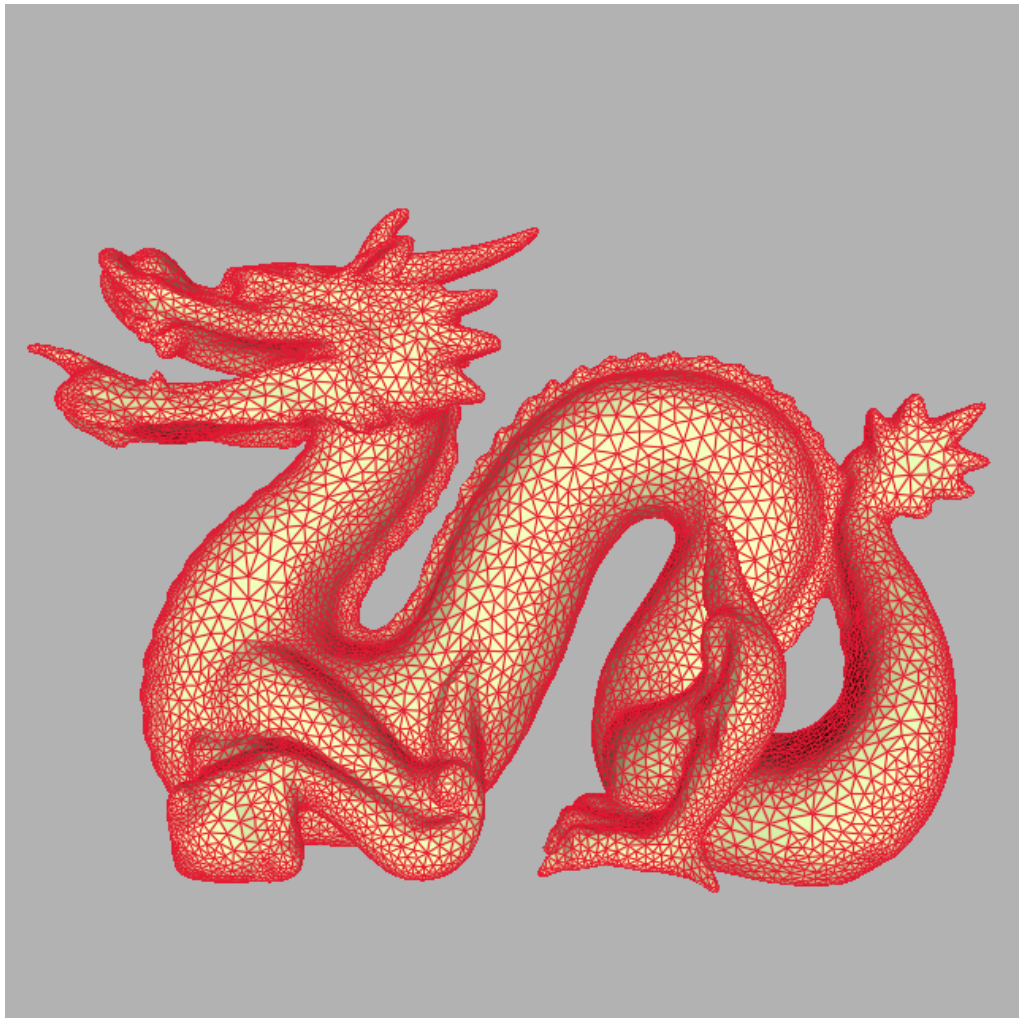

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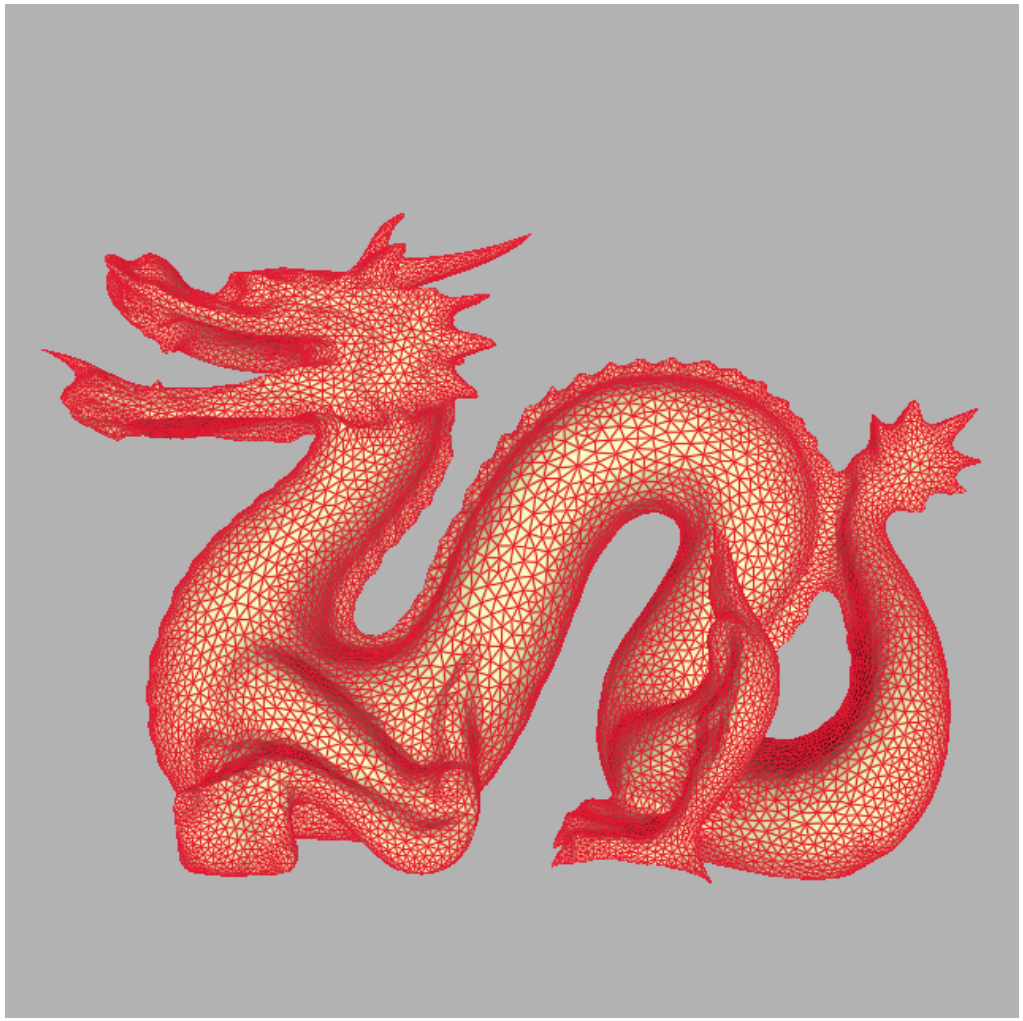

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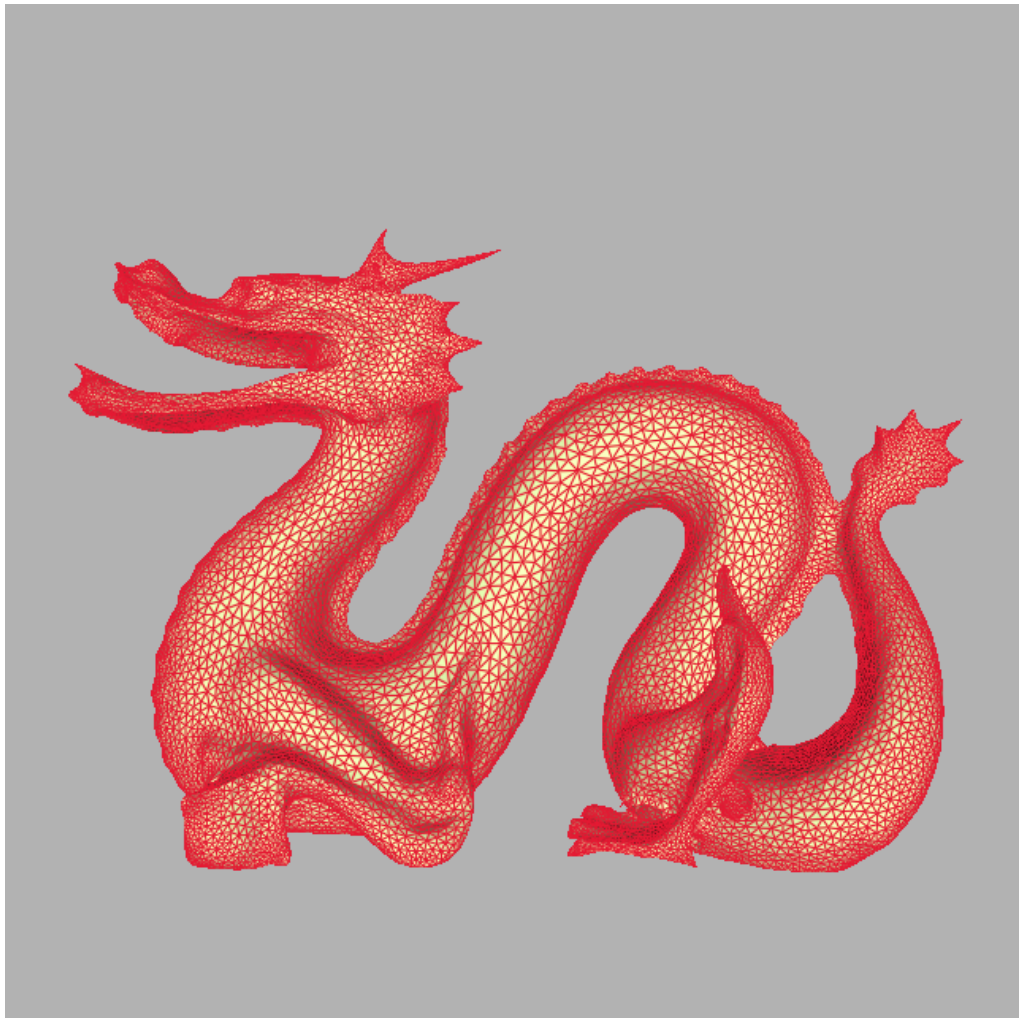

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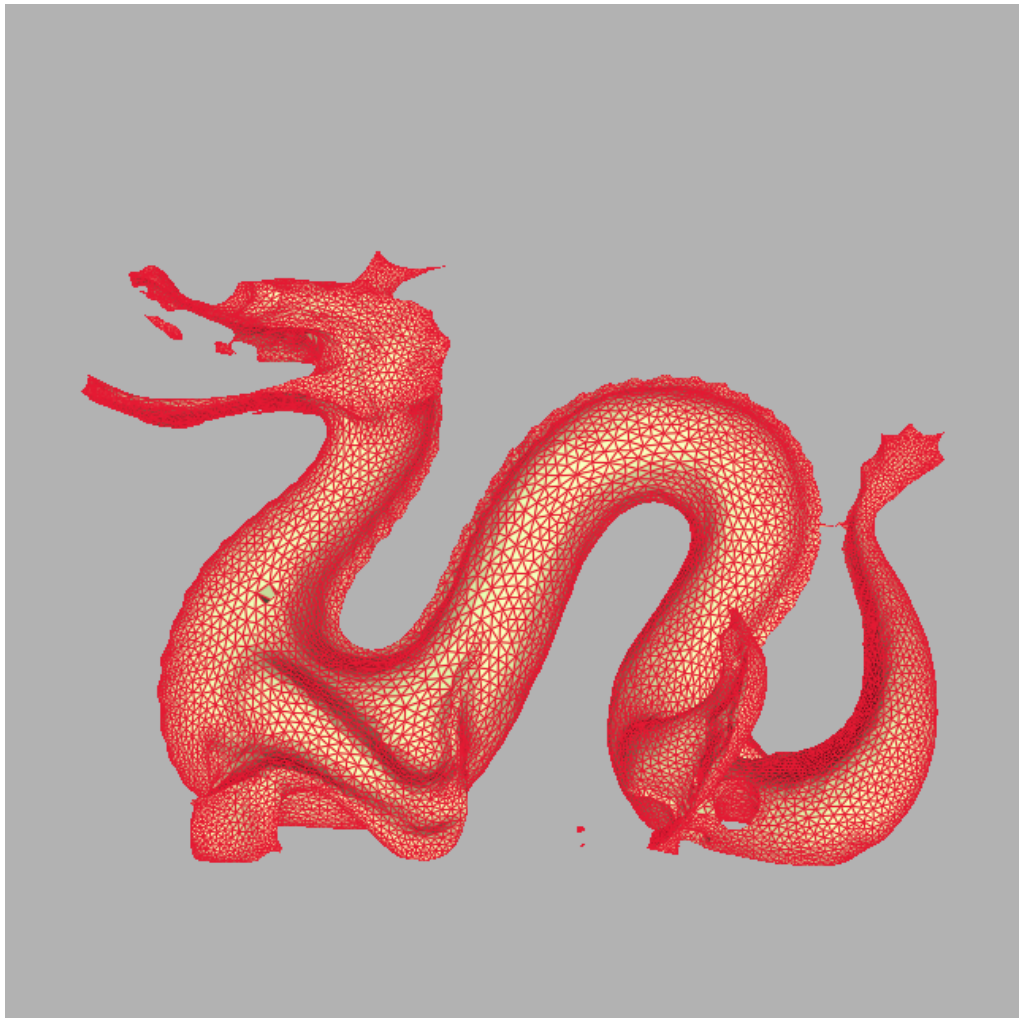

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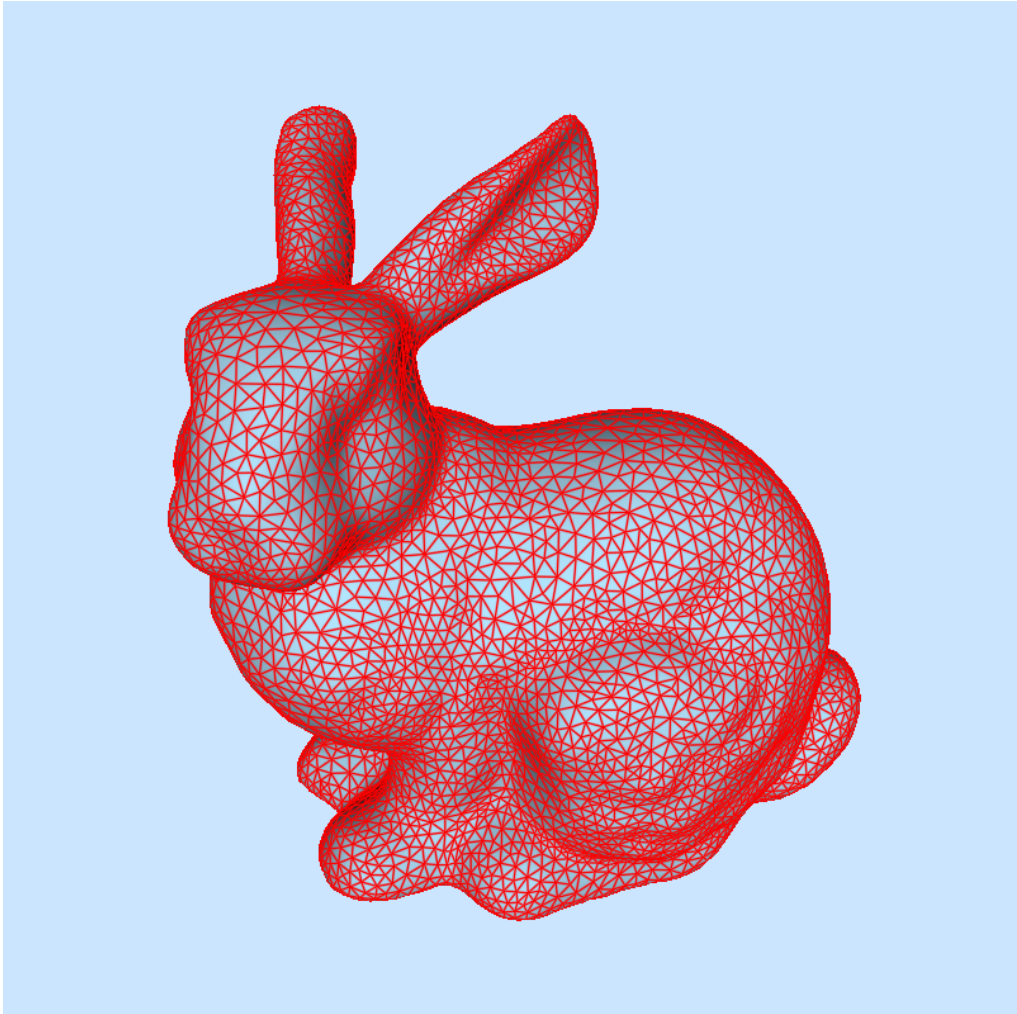

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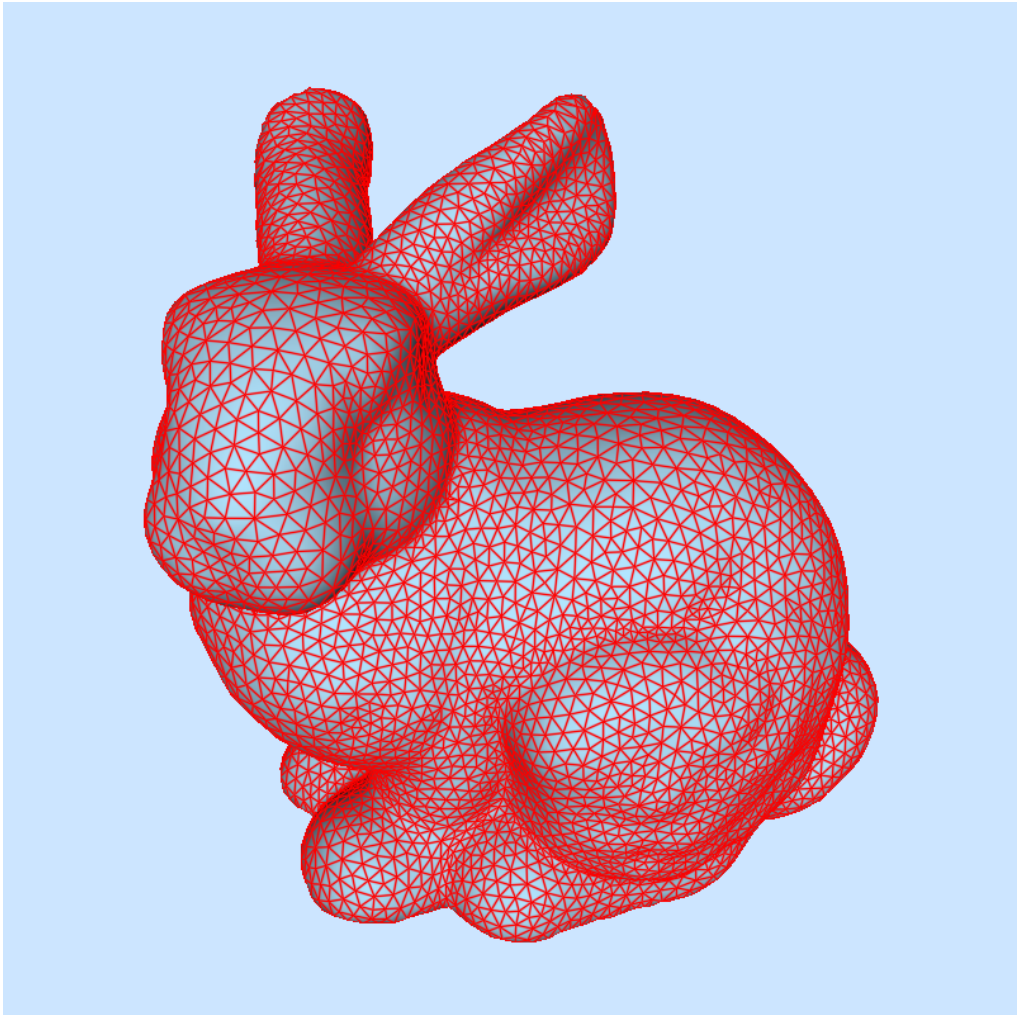

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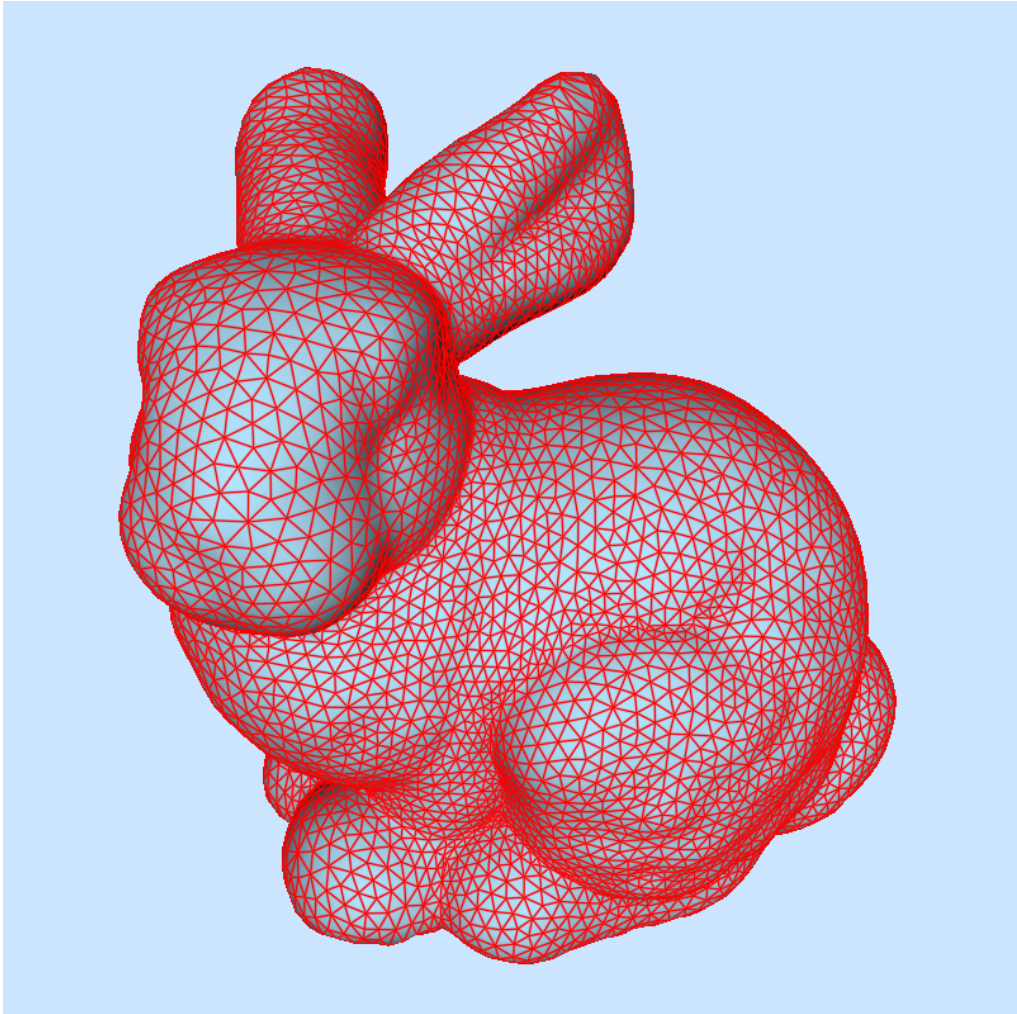

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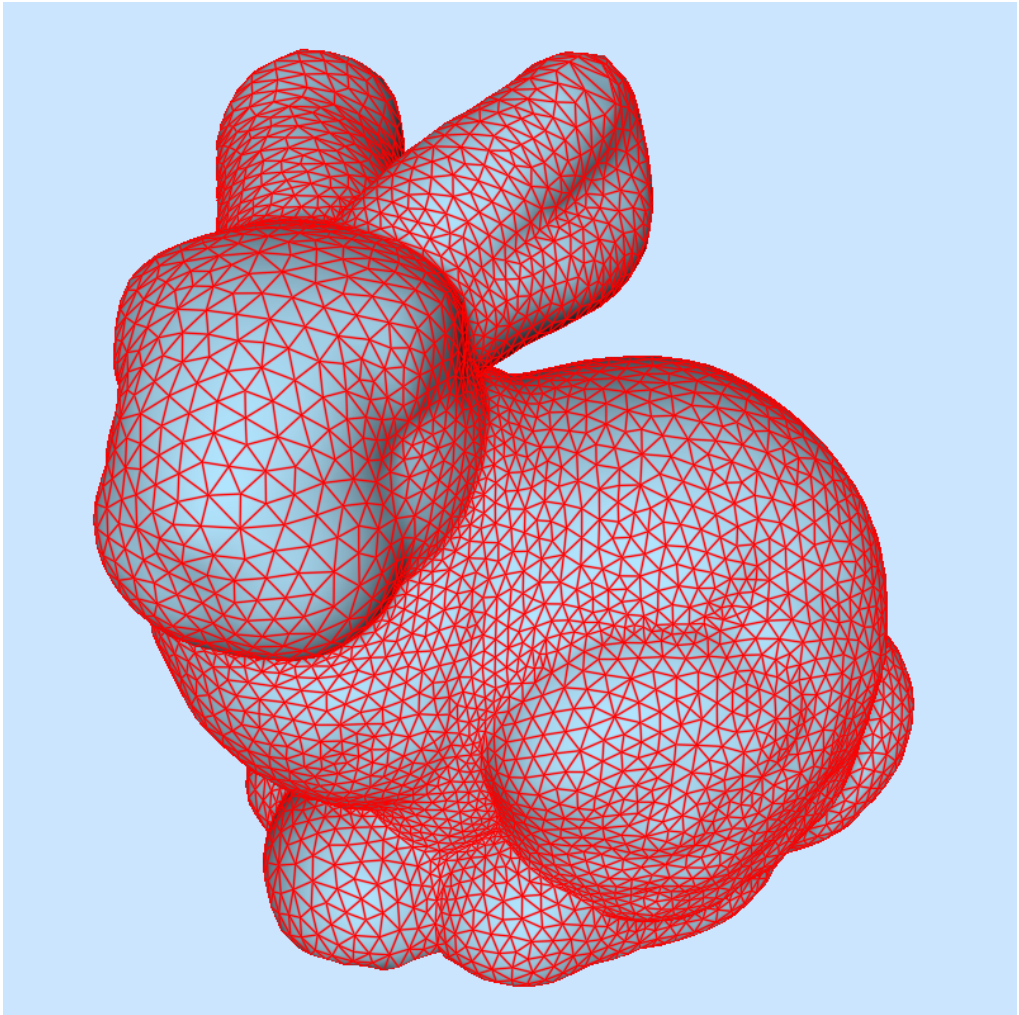
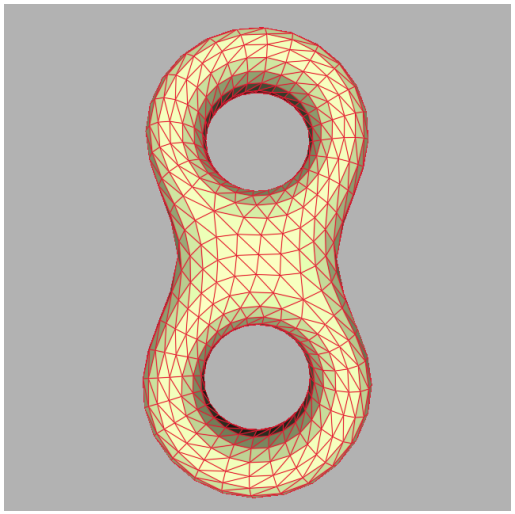
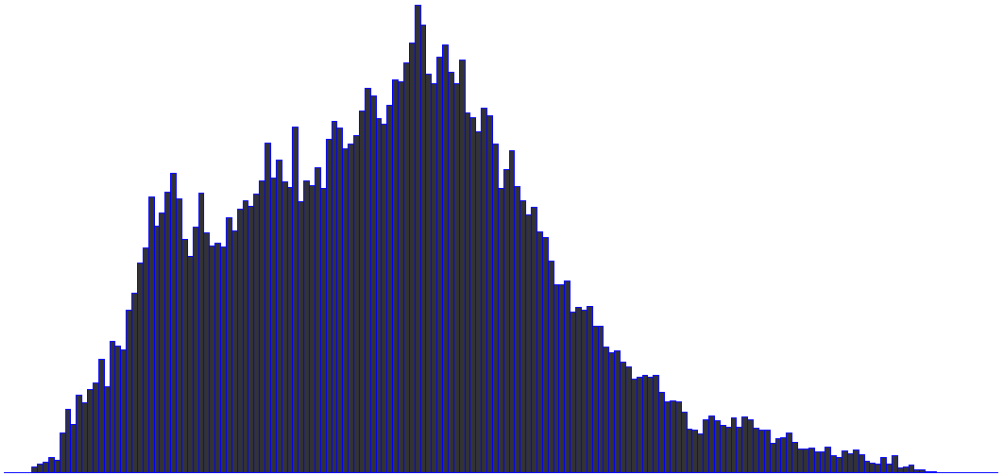


Performance

example	GENUS2	2SPHERES	BUNNY	DRAGON
#surf. vertices	766	1680	6929	27549
#surf. triangles	1536	3348	13934	55098
#total tets	10309	16433	77631	372015
#inside tets	2703	5554	35362	160110
time per iteration	1.2 s	1.4 s	18.2 s	48.0 s

Dihedral angles





Although low quality tetrahedra occasionally appear in the DSC mesh, dihedral angles from outside 6°–171° range constitute less than 0.1% of all dihedral angles in the tetrahedral mesh throughout all iterations.

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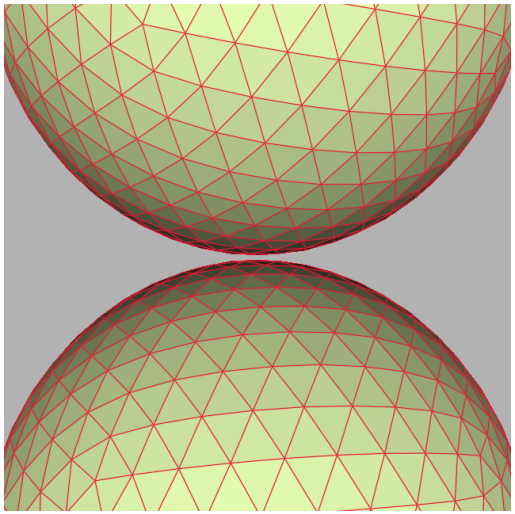

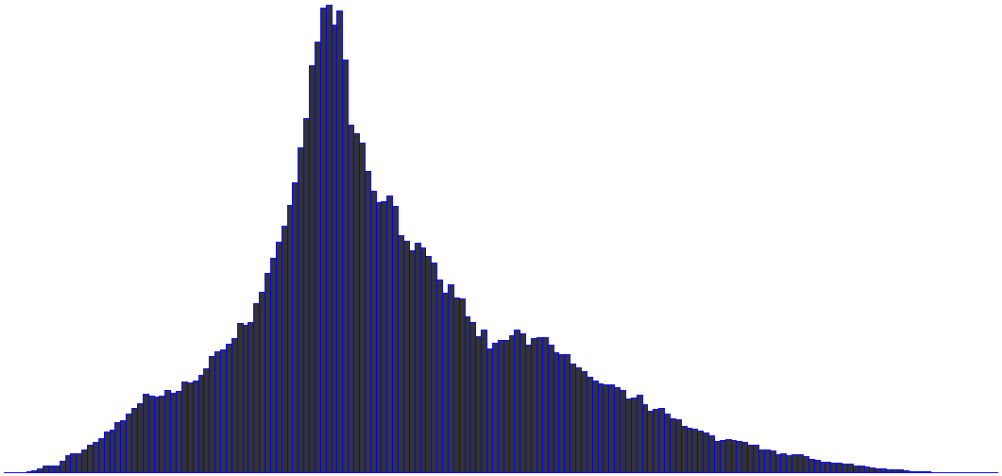

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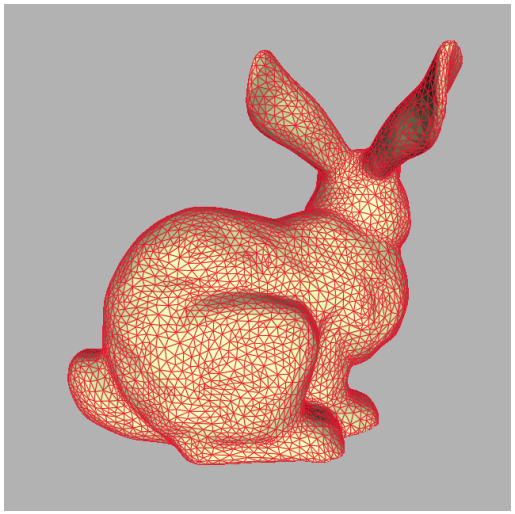

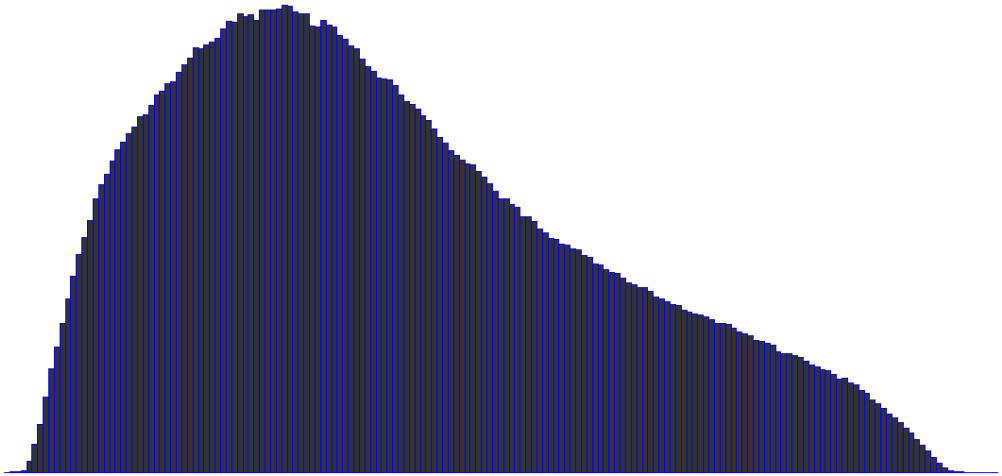

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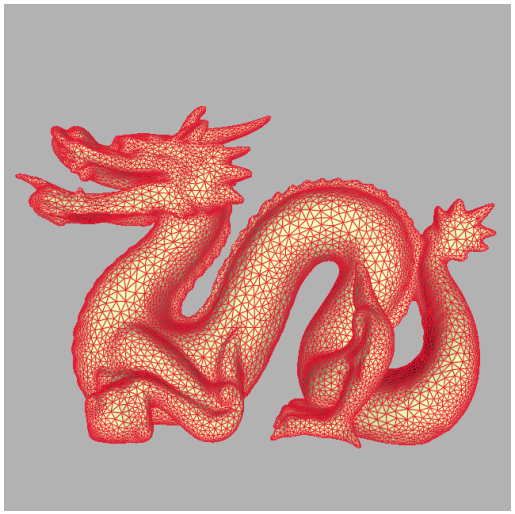

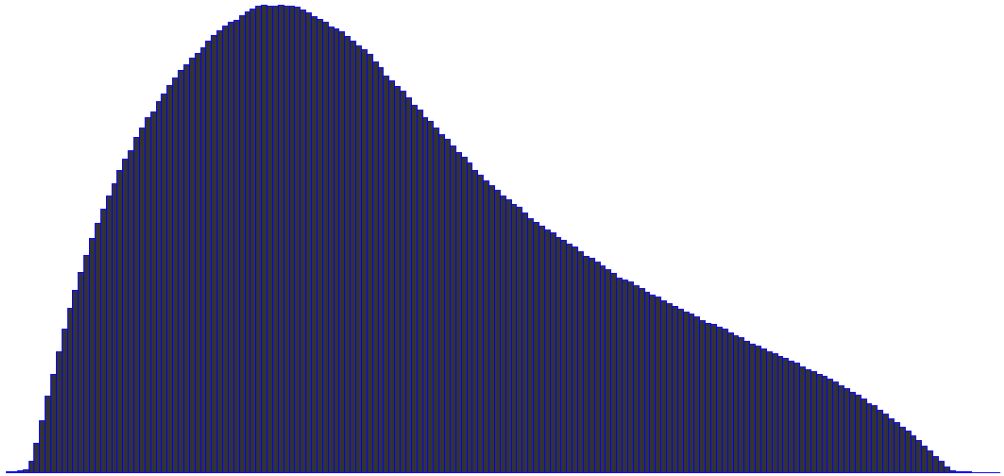

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